NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

A STABILITY ANALYSIS
OF THE PROPOSED
CIRCULATION CONTROL ROTOR (CCR) PROTOTYPE

by

John Hendrix Cline

March 1977

Thesis Advisor: L. V. Schmidt

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A Stability Analysis of the Proposed Circulation Control Rotor (CCR) Prototype		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis; March 1977
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) John Hendrix Cline		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		12. REPORT DATE March 1977
		13. NUMBER OF PAGES 60
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Circulation Control Rotor (CCR)
XH-2/CCR
BASMAT

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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(20. ABSTRACT Continued)

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A Stability Analysis of the Proposed Circulation Control Rotor (CCR) Prototype

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Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN AERONAUTICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
March 1977

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The rotor system of the proposed XH-2/CCR (Circulation Control Rotor) prototype aircraft and the state variable format of the airframe equations of motion are described. Through a study of the eigenvalues and eigenvectors of the basic airframe, the effects of uncoupling and cross-coupling the helicopter equations of motion were analyzed. The control matrix for the CCR was then generated using an applicable pneumatic lead angle for Coanda blowing. Feedback gains, to give the aircraft acceptable flying qualities, were calculated for the Stability Augmentation System (SAS) and finally the aircraft's sensitivity to changes in pneumatic lead angles were studied. The programs used in the above study (capable of handling up to 10 x 10 matrices) are listed for both the IBM-360 and HP-9830 mini-computer.

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LIST OF SYMBOLS AND ABBREVIATIONS

g	Acceleration due to gravity
L	Rolling moment about the x-axis due to aerodynamic torques
M	Pitching moment about the y-axis due to aerodynamic torques
N	Yawing moment about the z-axis due to aerodynamic torques
p	Roll rate, angular velocity about the x-axis (positive right wing down) $p = \phi$
q	Pitch rate, angular velocity about y-axis (positive nose up) $q = \dot{\theta}$
r	Yaw rate, angular velocity about z-axis (positive nose right)
u	Linear perturbation velocity along x-axis (positive forward)
U	Linear steady-state velocity along the x-axis (positive forward)
v	Linear perturbation velocity along y-axis (positive out right wing)
w	Linear perturbation velocity along z-axis (positive down)
X	Aerodynamic force along x-axis (positive forward)
Y	Aerodynamic force along y-axis (positive out right wing)
Z	Aerodynamic force along z-axis (positive down)
ϕ	Phase angle of control system
ψ	Blade aximuth angle from aft position

I. INTRODUCTION

In the early 1970's the David W. Taylor Naval Ship Research and Development Center (DTNSRDC) initiated a research program into the feasibility of incorporating a Circulation Controlled Rotor (CCR) system in a Navy helicopter. The concept of the CCR and of improving airfoil lift-to-drag ratios using Coanda flows had been proven earlier by some of the world's leading aerodynamicist's and as early as 1959 Dorand had published in the Journal of the Helicopter Association of Great Britain an article on the application of a jet flap to control a helicopter rotor (Ref. 1). Studies and tests continued throughout the 1960's with more papers published in both the United States and in Europe on the improved performance and possible applications of a CCR (Ref. 2).

The Aviation and Surface Effects Division of DTNSRDC continued the research with further tests involving detailed pressure measurements of two-dimensional elliptical sections in their 15 x 20-inch subsonic wind tunnel. These tests reconfirmed that extremely high lift-to-drag ratios could be achieved by tangentially injecting air through a slot in the trailing edge of an airfoil. The results of these tests were incorporated in both two- and four-bladed model rotor systems for evaluation in the DTNSRDC 8 x 10 foot windtunnel (Ref. 3).

The incorporation of a CCR system in a full-size aircraft could conceivably offer other advantages over the conventional rotor system, in addition to the potentially improved aerodynamic performance traits. The conventional rotating mechanical swashplate system would be replaced by a non-rotating pneumatic plate in a plenum chamber located in the blade hub region. Collective control would be accomplished by changing the plenum chamber pressure, which increases or decreases the Coanda blowing equally at all blades via the individual blade supply or collector tubes. Cyclic control would be provided by tilting the pneumatic swashplate so that there is an azimuthal variation in Coanda blowing in each blade. This variation in blowing is a result of the changes in volume of air allowed to the collector tubes because of changes in the gap between the collector tubes and the non-rotating swashplate. This non-rotating swashplate and variations in Coanda blowing would replace the mechanical cyclic feathering required by conventional rotors and therefore eliminate the vibrations caused by this one-per-revolution cyclic mechanical movement. Another important point to recognize is that the CCR concept, with the Coanda blowing, will dictate a torsionally stiff rotor blade or rigid rotor system. This is a result of the disparity between the two lift generation centroids. The center of pressure due to Coanda blowing is near the blade midchord region, while the blade aerodynamic center remains near the rotor blade quarter chord point.

The proposed helicopter, with a simple hub and rigid rotor blades free of flapping and lag hinges, would result in a relatively clean aerodynamic hub system. The reduction in rotor and hub drag would be beneficial to the helicopter from a performance standpoint. The reduction in moving parts in the hub and blade system would also mean a quieter helicopter with a lower vibration level than that of a conventional rotor system. This latter effect has a favorable potential of improving the "ilities" (maintainability and reliability) for helicopter operations.

II. BACKGROUND

Early in 1973 the Naval Air Systems Command (NAVAIR) contracted the Lockheed Aircraft Company and Kaman Aerospace Corporation to investigate the feasibility of developing a full-scale flightworthy Circulation Controlled Rotor demonstrator aircraft. In the summer of 1974, some twelve months later, both companies returned reports to DTNSRDC and NAVAIR stating that: "the concept, while innovative, is completely safe in operation" (Ref. 4) and "that there is no fundamental flaw or deficiency in the CCR concept and that construction of a full scale CCR helicopter is feasible and practical" (Ref. 5). Lockheed Aircraft proposed the use of its L286/CCR while Kaman suggested "that the Kaman/Navy H-2 aircraft is an ideal test vehicle for the CCR concept" (Ref. 4).

With the additional goal of being able to incorporate a CCR system on any off-the-shelf helicopter with no major airframe or equipment changes, NAVAIR awarded a contract to Kaman Aerospace to "develop, build and test" a prototype CCR vehicle incorporating the use of the Navy/Kaman H-2 aircraft. This technology demonstration aircraft will tentatively be designated as the Navy XH-2/CCR.

Preliminary studies by Kaman promoted the belief that acceptable flying qualities would be sustained with the installed Stability Augmentation System (SAS) of the Kaman

H-2 with only minor changes in the gains of the feedback amplifiers (Ref. 5). Acceptable flying qualities does not necessarily mean all stable roots of the aircraft motion modes, since a weak oscillatory instability with a time-to-double amplitude of greater than three seconds can be tolerated by a proficient rotor-wing aircraft pilot. The objectives of this research was to confirm that Kaman's beliefs were in fact true and to find a suitable feedback law for the SAS of the XH-2/CCR such that within the aircraft's flight envelope the aircraft flying qualities will be acceptable to the evaluation pilot.

III. DISCUSSION

The study of the helicopter flight dynamics were conducted using the conventional non-dimensionalized state variable format of the aircraft linearized equations of motion (Ref. 6), modified to allow coupling of the longitudinal with the lateral-directional motions. This modification is a fairly elementary record-keeping operation when using state vector formulations. The basic plant matrix, A, of the aircraft linearized equations of motion in the state variable format is given in Appendix A.

The stability derivatives for the XH-2/CCR airframe were computer generated by the contractor using the MOSTAB-HFA program (Ref. 7) modified for the pertinent characteristics of the SH-2F airframe and the XH-2/CCR main rotor system. The flight conditions analyzed were for 1.0g level flight at sea level standard conditions. The aircraft gross weight was given as 11,000 pounds and a rotor tip speed of 615 feet per second (267 RPM) was used throughout the calculations. Stability derivatives were generated for airspeeds of: Zero (hover), 35, 72, 110, and 130 knots. These derivatives were computed in May of 1976 and then updated in November of the same year. The calculations made in this research effort are based on the updated, November 1976, data.

A. THE PLANT MATRIX

The plant matrix, A, was developed for the longitudinal and lateral-directional components and then the fully-coupled equations of motion using the contractor generated stability derivatives. The plant matrix, A, was partitioned into:

$$A = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right]$$

where $[A_{11}]$ represented the coefficients of the longitudinal stability derivatives and $[A_{22}]$ represented the coefficients of the lateral-directional stability derivatives. The cross-coupling stability derivatives were represented by the coefficients of $[A_{12}]$ and $[A_{21}]$.

The homogeneous form of the state equations took on the conventional form of:

$$\dot{\underset{\sim}{x}} = \left\{\begin{array}{c} \dot{x}_1 \\ \hline \dot{x}_2 \end{array}\right\} = A\,\underset{\sim}{x} = A\left\{\begin{array}{c} x_1 \\ \hline x_2 \end{array}\right\}$$

where the state vector, x, included the partitioned longitudinal airframe state vector:

$$x_1^T = [u,\ w,\ q,\ \theta]$$

and the lateral-directional airframe state vector respectively:

$$x_2^T = [p, r, v, \phi]$$

Normally in airframe dynamics the fact that most "fixed wing" aircraft approach symmetric conditions enables one to eliminate the cross coupling terms $[A_{12}]$ and $[A_{21}]$ and then to represent the aircraft by the decoupled equations of motion in the longitudinal and lateral-directional modes. Neglecting these cross-coupling terms allows analysis of the individual fourth-order systems and the size and shape of the applicable longitudinal and lateral-directions modes changes only slightly when the more complicated computations are made for the fully coupled eighth-order systems. Unfortunately, helicopters do not enjoy these conditions of symmetry and the effects of completely cross-coupling the longitudinal and lateral-directional equations are quite significant.

The fact that the helicopter had large cross-coupling effects was shown by the significant change in the eigenvalues for the uncoupled and fully-coupled computations and modal identification could not be completed from the results of the cross-coupled forms alone. The conclusion, therefore, was that the stability problem could not be solved by studying only the uncoupled components of the

equations of motion but must be undertaken using the fully coupled, eighth-order equations of motion.

Using the Basic Matrix Control Theory (BASMAT) computer program (Ref. 8) modified for the HP-9830 mini-computer and the IBM-360 digital computer located at the Naval Postgraduate School's W. R. Church Computer Center; calculations were made at the above stated airspeeds and the eigenvalues and eigenvectors of the plant matrix, A, were obtained for both the uncoupled and fully-coupled equations of motion. The revised programs for both the IBM-360 and the HP-9830 are included as part of this paper in Appendix B and C respectively.

The existence of a longitudinal unstable root in the uncoupled form was confirmed at all calculated speeds and the identification of the different modes in the fully-coupled eighth-order system was completed by slowly introducing the cross-coupling derivatives into the aircraft equations of motion. The eigenvalues and eigenvectors were traced and modes identified by letting A_{21} and A_{12} equal zero in the eighth-order system and then slowly increasing their values until they reached the final values of the basic plant matrix, A. The values of the uncoupled eigenvalues from the fourth-order solutions and the final eighth-order system results are included in Table I.

A modified root-locus is shown in Figures 1 and 2 for the airspeed conditions of hover and 130 knots respectively. The trajectory of the roots is shown as the amount

TABLE I

Eigenvalues of Basic XH-2/CCR Airframe

Note: Uncoupled airframe listing shows four longitudinal roots first followed by four lateral-directional roots.

Hover (uncoupled)	(coupled)
$\lambda = 0.00159 \pm i\ 0.10077$	$\lambda = -0.28206 \pm i\ 0.01601$
$= -0.23713$	$= 0.01867$
$= -2.64654$	$= -0.05189$
$= 0.02949 \pm i\ 0.27626$	$= -0.00634 \pm i\ 0.20903$
$= -0.44778$	$= -5.18781 \pm i\ 2.53260$
$= -7.71637$	

35 Knots (uncoupled)	(coupled)
$\lambda = -0.00971 \pm i\ 0.16470$	$\lambda = 0.01046 \pm i\ 0.13193$
$= -0.30077$	$= -0.24564$
$= -2.37101$	$= -4.50717 \pm i\ 2.34599$
$= -0.24771 \pm i\ 0.81760$	$= -0.27651 \pm i\ 0.85642$
$= -0.05339$	$= -0.06194$
$= -6.61402$	

72 Knots (uncoupled)	(coupled)
$\lambda = -0.79763$	$\lambda = -0.05862 \pm i\ 0.15912$
$= -0.14599$	$= -0.27382$
$= -0.11719$	$= -4.40993 \pm i\ 2.33811$
$= -2.19846$	$= -0.42956 \pm i\ 1.53113$
$= -0.41632 \pm i\ 1.42319$	$= -0.05781$
$= -0.05296$	
$= -6.10178$	

110 Knots (uncoupled)	(coupled)
$\lambda = -0.38109 \pm i\ 0.26126$	$\lambda = -0.15433 \pm i\ 0.25617$
$= 0.31503$	$= 0.36151$
$= -2.92894$	$= -4.52643 \pm i\ 2.93431$
$= -0.54931 \pm i\ 1.97156$	$= -0.53360 \pm i\ 2.10332$
$= -0.04699$	$= -0.04816$
$= -5.59397$	

130 Knots (uncoupled)	(coupled)
$\lambda = -0.30712 \pm i\ 0.29749$	$\lambda = -0.07151 \pm i\ 0.20758$
$= 0.31242$	$= 0.41211$
$= -3.02559$	$= -4.35708 \pm i\ 2.81962$
$= -0.63733 \pm i\ 2.22591$	$= -0.56744 \pm i\ 2.34661$
$= -0.05636$	$= -0.08227$
$= -5.00383$	

FIGURE 1. CROSS-COUPLING EFFECTS UPON AIRFRAME MODES (HOVER)

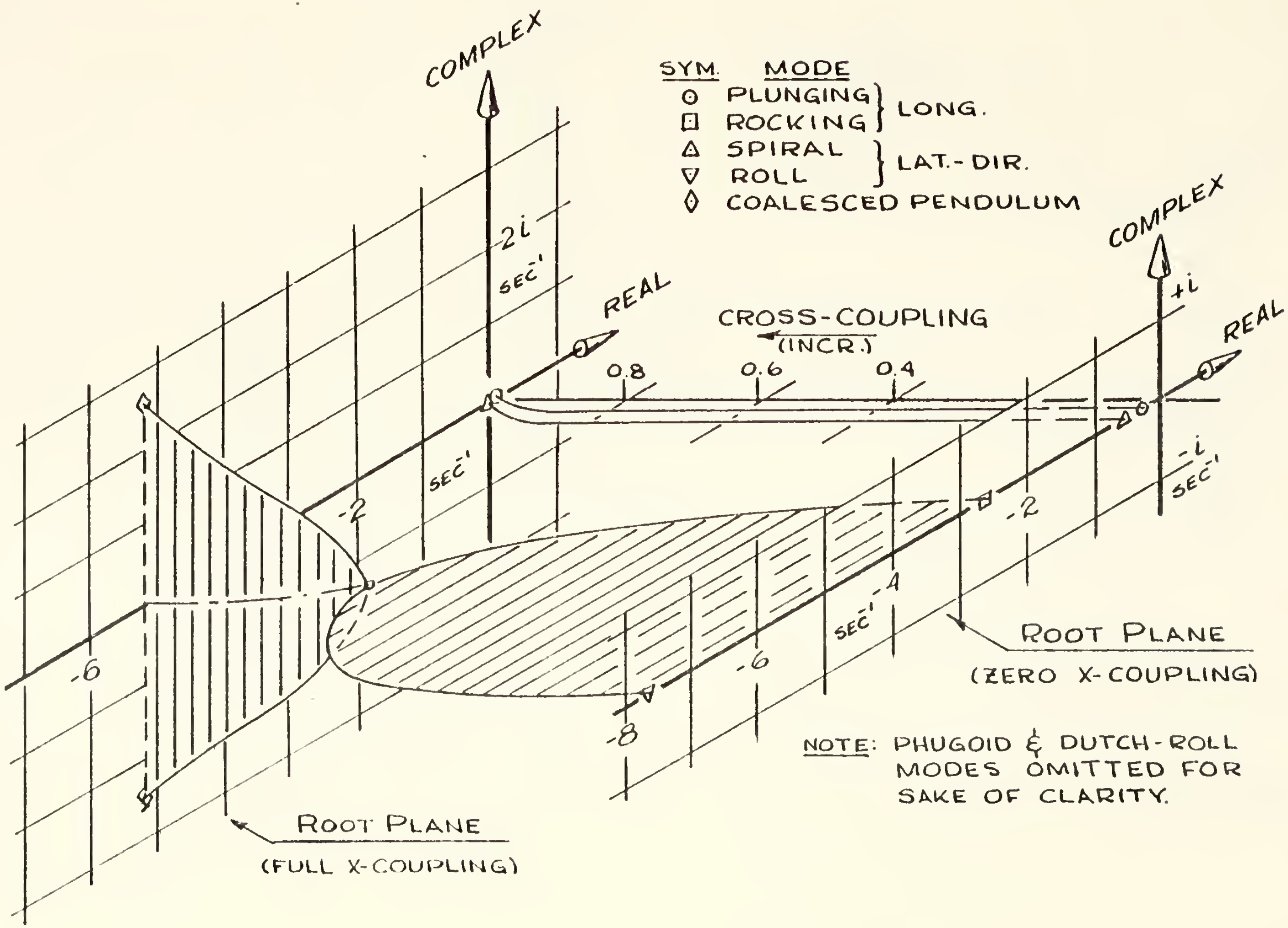
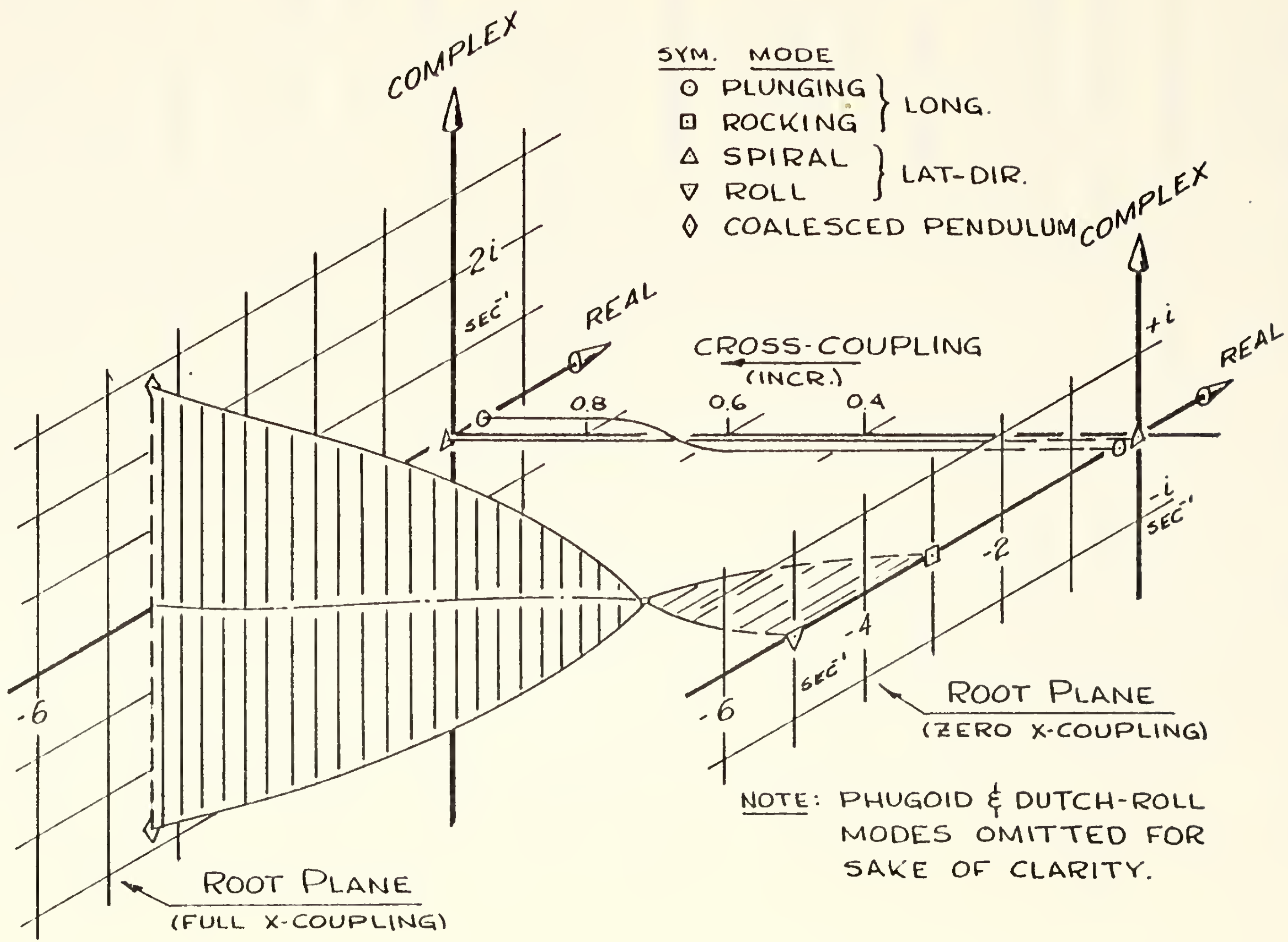


FIGURE 2. CROSS-COUPLING EFFECTS UPON AIRFRAME MODES (U = 130 KNOTS)



of cross-coupling increased from 0.0 to 1.0 with the latter limit corresponding to a fully cross-coupled system. The Dutch-roll and long period oscillatory roots are omitted for sake of clarity, but their values do not vary significantly with cross-coupling as shown in Table I.

The uncoupled non-oscillatory lateral-directional roots may be identified as spiral and roll subsidence roots by both the mode shape and time constants using familiar analogies from fixed wing aircraft. The remaining two non-oscillatory roots from the uncoupled longitudinal degrees of freedom would normally correspond to a short period situation in fixed wing aircraft.

It was observed that the low time constant-real roots, one each from the longitudinal and lateral-directional degrees of freedom respectively remain almost invariant with the amount of cross-coupling until they reach the neighborhood of full cross-coupling. At that time the longitudinal root becomes weakly unstable with a time-to-double amplitude of approximately 37 and 1.68 seconds at hover and 130 knots respectively. The latter situation definitely required improvement by means of stability augmentation. These two real roots could have been expected to coalesce into an oscillatory pair as cross-coupling varied, but possibly the close proximity to the almost invariant oscillatory long period and Dutch-roll roots prevented this action from occurring.

The second real-longitudinal root and the roll subsidence root may be observed to coalesce into a pair of complex conjugate (oscillatory roots) almost mid-range in the cross-coupling. The coalesced roots have a mode shape similar to a pendulum type of motion, but it was noted that this mode (in the fully-coupled situations) was quite heavily damped.

The aircraft mode shapes are defined in Table II for hover and 130 knots velocities in both the uncoupled and fully coupled situations. Without these mode shapes, there would be difficulties involved in interpreting the Characteristic root migrations as shown in Figures 1 and 2. The symbol (⊙) is used on the figures to identify a longitudinal plunging subsidence mode which will be spotted in Table II with the (w) velocity perturbation being the dominant term. This root, which in conventional airframe systems would be combined with the rocking mode root (⊡) to yield the conventional oscillatory short period mode, is the "culprit" which becomes unstable in the fully coupled situation. As will be noted in Figures 1 and 2, the longitudinal rocking and the lateral roll subsidence modes coalesce into an oscillatory pendulum type mode.

B. THE CONTROL MATRIX

In addition to the definition of the airframe plant, the contractor provided the MOSTAB generated control matrices. The derivative information, presumably the result

TABLE II

MODE SHAPE SUMMARY

A. Hover Conditions

1. Zero Cross-Coupling

a. Longitudinal-oscillatory long period

$\lambda = 0.0016 \pm i\ 0.1008$ Period = 62.3 sec T_2 = 435.94 sec.

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \end{Bmatrix} = \begin{Bmatrix} 1.0000; & \text{arg.} & 0.0\ \text{deg.} \\ 0.0196; & \text{arg.} & -116.8\ \text{deg.} \\ 0.0003; & \text{arg.} & 83.8\ \text{deg.} \\ 0.0031; & \text{arg.} & -\ 5.4\ \text{deg.} \end{Bmatrix}$$

b. Longitudinal-plunging subsidence

$\lambda = -0.2371$ $T_{\frac{1}{2}}$ = 2.92 sec.

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0.0700 \\ 1.0000 \\ 0.0000 \\ 0.0000 \end{Bmatrix}$$

c. Longitudinal-fore/aft rocking subsidence

$\lambda = -2.647$ $T_{\frac{1}{2}}$ = 0.26 sec.

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \end{Bmatrix} = \begin{Bmatrix} 1.0000 \\ -0.0090 \\ -0.1830 \\ 0.0690 \end{Bmatrix}$$

d. Lateral-Directional-Dutch Roll

$\lambda = 0.0295 \pm i\ 0.2762$ Period = 22.8 sec. T_2 = 23.4 sec.

$$\begin{Bmatrix} p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 0.0025; & \text{arg.} & 166.4\ \text{deg.} \\ 0.0250; & \text{arg.} & -\ 34.4\ \text{deg.} \\ 1.0000; & \text{arg.} & 0.0\ \text{deg.} \\ 0.0090; & \text{arg.} & 82.4\ \text{deg.} \end{Bmatrix}$$

e. Directional-Spiral subsidence

$\lambda = -0.448$ $T_{\frac{1}{2}}$ = 1.55 sec.

$$\begin{Bmatrix} p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 0.0044 \\ -0.1650 \\ 1.000 \\ 0.0099 \end{Bmatrix}$$

f. Lateral-Roll Subsidence

$\lambda = -7.716$ $T_{\frac{1}{2}}$ = 0.090 sec.

$$\begin{Bmatrix} p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 1.0000 \\ -0.0294 \\ 0.8669 \\ -0.1296 \end{Bmatrix}$$

TABLE II (Continued)

A. 2. Full Cross-coupling

a. Longitudinal-oscillatory long period

$\lambda = -0.2821 \pm i\ 0.0160$ Period = 392.7 sec. $T_{\frac{1}{2}}$ = 2.46 sec

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \\ p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 1.0000;\ \text{arg.}\ 0.0\ \text{deg.} \\ 0.7102;\ \text{arg.}\ 14.1\ \text{deg.} \\ 0.0025;\ \text{arg.}\ 173.4\ \text{deg.} \\ 0.0089;\ \text{arg.}\ -\ 3.3\ \text{deg.} \\ 0.0008;\ \text{arg.}\ 176.0\ \text{deg.} \\ 0.0490;\ \text{arg.}\ 174.5\ \text{deg.} \\ 0.1924;\ \text{arg.}\ 182.6\ \text{deg.} \\ 0.0028;\ \text{arg.}\ -\ 0.7\ \text{deg.} \end{Bmatrix}$$

b. Cross-coupled Pendulum Mode

$\lambda = -5.1878 \pm i\ 2.5326$ Period = 2.48 sec. $T_{\frac{1}{2}}$ = 0.13 sec

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \\ p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 0.6181;\ \text{arg.}\ -141.88\ \text{deg.} \\ 0.0798;\ \text{arg.}\ 155.40\ \text{deg.} \\ 0.4219;\ \text{arg.}\ 2.14\ \text{deg.} \\ 0.0731;\ \text{arg.}\ -155.84\ \text{deg.} \\ 0.7618;\ \text{arg.}\ -\ 42.92\ \text{deg.} \\ 0.0386;\ \text{arg.}\ 177.95\ \text{deg.} \\ 1.0000;\ \text{arg.}\ 0.0\ \text{deg.} \\ 0.1320;\ \text{arg.}\ 163.10\ \text{deg.} \end{Bmatrix}$$

c. Lateral-Direction - Coupled Dutch Roll Mode

$\lambda = -0.0063 \pm i\ 0.2090$ Period = 30.1 sec. $T_{\frac{1}{2}}$ = 109.26 sec.

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \\ p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 1.0000;\ \text{arg.}\ 0.00\ \text{deg.} \\ 0.0162;\ \text{arg.}\ -171.87\ \text{deg.} \\ 0.0014;\ \text{arg.}\ 3.04\ \text{deg.} \\ 0.0065;\ \text{arg.}\ -\ 88.70\ \text{deg.} \\ 0.0006;\ \text{arg.}\ 145.88\ \text{deg.} \\ 0.0126;\ \text{arg.}\ 1.69\ \text{deg.} \\ 0.4099;\ \text{arg.}\ 33.20\ \text{deg.} \\ 0.0028;\ \text{arg.}\ 122.38\ \text{deg.} \end{Bmatrix}$$

d. Lateral-

$\lambda = -0.0519$ $T_{\frac{1}{2}}$ = 13.36 sec.

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \\ p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 1.0000 \\ -0.0358 \\ -0.0000 \\ 0.0016 \\ -0.0000 \\ -0.0077 \\ -0.1992 \\ 0.0003 \end{Bmatrix}$$

e. Longitudinal

$\lambda = 0.0187$ $\qquad T_2 = 37.13$ sec.

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \\ p \\ r \\ \phi \end{Bmatrix} = \begin{Bmatrix} 1.0000 \\ -0.0224 \\ -0.0000 \\ -0.0006 \\ -0.0000 \\ -0.2016 \\ -0.0001 \end{Bmatrix}$$

B. 130 Knots

1. Zero Cross-coupling

a. Longitudinal-oscillatory Long Period Mode

$\lambda = -0.3071 \pm i\ 0.2975$ Period = 21.1 sec. $T_{\frac{1}{2}} = 2.26$ sec.

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0.5812;\ \text{arg.}\ -67.70\ \text{deg.} \\ 1.0000;\ \text{arg.}\ 0.0\ \text{deg.} \\ 0.0026;\ \text{arg.}\ 39.45\ \text{deg.} \\ 0.0061;\ \text{arg.}\ -96.46\ \text{deg.} \end{Bmatrix}$$

b. Longitudinal -

$\lambda = 0.3124$ $\qquad T_2 = 2.22$ sec.

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \end{Bmatrix} = \begin{Bmatrix} 1.0000 \\ -0.7721 \\ -0.0043 \\ -0.0138 \end{Bmatrix}$$

c. Longitudinal - $\qquad T_{\frac{1}{2}} = 0.23$ sec.

$\lambda = -3.0256$

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0.0185 \\ 1.0000 \\ -0.0108 \\ 0.0036 \end{Bmatrix}$$

d. Lateral-Directional Dutch Roll Mode

$\lambda = -0.6373 \pm i\ 2.2259$ Period = 2.8 sec. $T_{\frac{1}{2}} = 1.09$ sec.

$$\begin{Bmatrix} p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 0.0185;\ \text{arg.}\ 0.02\ \text{deg.} \\ 1.0000;\ \text{arg.}\ 0.0\ \text{deg.} \\ 0.0107;\ \text{arg.}\ 179.96\ \text{deg.} \\ 0.0036;\ \text{arg.}\ -\ 0.0\ \text{deg.} \end{Bmatrix}$$

e. Lateral-

$\lambda = -0.0564$ $\qquad T_{\frac{1}{2}} = 12.29$ sec.

$$\begin{Bmatrix} p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} -0.0077 \\ 0.0203 \\ 1.0000 \\ 0.1363 \end{Bmatrix}$$

f. Lateral-

$\lambda = -5.0038$ $\qquad T_{\frac{1}{2}} = 0.14$ sec.

$$\begin{Bmatrix} p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 1.0000 \\ -0.0381 \\ -0.0674 \\ -0.1999 \end{Bmatrix}$$

B. 2. Full Cross-coupling

a. Longitudinal-oscillatory Long Period Mode

$\lambda = -0.0715 \pm i\ 0.2076$ $\quad$ Period = 30.3 sec. $\quad T_{\frac{1}{2}} = 9.69$ sec.

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \\ p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 1.0000;\ \text{arg.}\ \ 0.0\ \text{deg.} \\ 0.4987;\ \text{arg.}\ \ 10.05\ \text{deg.} \\ 0.0014;\ \text{arg.}\ \ 34.18\ \text{deg.} \\ 0.0064;\ \text{arg.}\ -74.83\ \text{deg.} \\ 0.0045;\ \text{arg.}\ \ 167.65\ \text{deg.} \\ 0.0027;\ \text{arg.}\ \ 81.22\ \text{deg.} \\ 0.3260;\ \text{arg.}\ \ 25.33\ \text{deg.} \\ 0.2051;\ \text{arg.}\ \ 883.34\ \text{deg.} \end{Bmatrix}$$

b. Lateral-Directional - Coupled Dutch Roll Mode

$\lambda = -0.5674 \pm i\ 2.3466$ $\quad$ Period = 2.7 sec. $\quad T_{\frac{1}{2}} = 1.22$ sec.

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \\ p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 0.0111;\ \text{arg.}\ -109.36\ \text{deg.} \\ 0.2045;\ \text{arg.}\ -20.66\ \text{deg.} \\ 0.0023;\ \text{arg.}\ -74.39\ \text{deg.} \\ 0.0009;\ \text{arg.}\ \ 88.99\ \text{deg.} \\ 0.0045;\ \text{arg.}\ \ 159.81\ \text{deg.} \\ 0.0110;\ \text{arg.}\ -76.86\ \text{deg.} \\ 1.0000;\ \text{arg.}\ \ 0.0\ \text{deg.} \\ 0.0019;\ \text{arg.}\ \ 56.22\ \text{deg.} \end{Bmatrix}$$

c. Cross-couple Pendulum Mode

$\lambda = -4.3571 \pm i\ 2.8196$ Period = 2.2 sec. $T_{\frac{1}{2}} = 0.16$ sec.

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \\ p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 0.0177;\ \text{arg.}\ \ 12.86\ \text{deg.} \\ 1.0000;\ \text{arg.}\ \ 0.0\ \text{deg.} \\ 0.0214;\ \text{arg.}\ -66.01\ \text{deg.} \\ 0.0041;\ \text{arg.}\ -3.95\ \text{deg.} \\ 0.0412;\ \text{arg.}\ \ 82.02\ \text{deg.} \\ 0.0066;\ \text{arg.}\ -8.08\ \text{deg.} \\ 0.2695;\ \text{arg.}\ \ 37.04\ \text{deg.} \\ 0.0079;\ \text{arg.}\ -65.07\ \text{deg.} \end{Bmatrix}$$

d. Longitudinal-

$\lambda = 0.4121$ $T_2 = 1.68$ sec.

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \\ p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} -0.7732 \\ 1.0000 \\ 0.0006 \\ 0.0014 \\ -0.0031 \\ -0.0014 \\ -0.0342 \\ -0.0075 \end{Bmatrix}$$

d. Lateral

$\lambda = -0.0823$ $T_{\frac{1}{2}} = 8.42$ sec.

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \\ p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 1.0000 \\ 0.0781 \\ -0.0000 \\ 0.0009 \\ -0.0041 \\ 0.0076 \\ 0.5428 \\ 0.0504 \end{Bmatrix}$$

of pneumo-dynamic modeling in the hub and blade blowing sections, was supplied as airframe forces and moments per unit (3060 psf) pressure variation at the pneumatic swashplate referenced to the aircraft axis. The cyclical variation of the plenum pressure upon control was given by:

C (2) .. The coefficient of the cosine ψ type variation in the control matrix.

C (3) .. The coefficient of the sine ψ type variation in the control matrix.

where, ψ, represents the blade azimuthal angle. One can visualize the longitudinal and lateral cyclic controls as rotating the pneumatic swashplate about an orthogonal set of axes that leads the blade azimuthal angle by some angle, ϕ, in an analogous manner to the practise on mechanical swashplates. Then, as shown in Figure 3, the apparent longitudinal and lateral cyclic control matrices become, by a coordinate rotation, as follows:

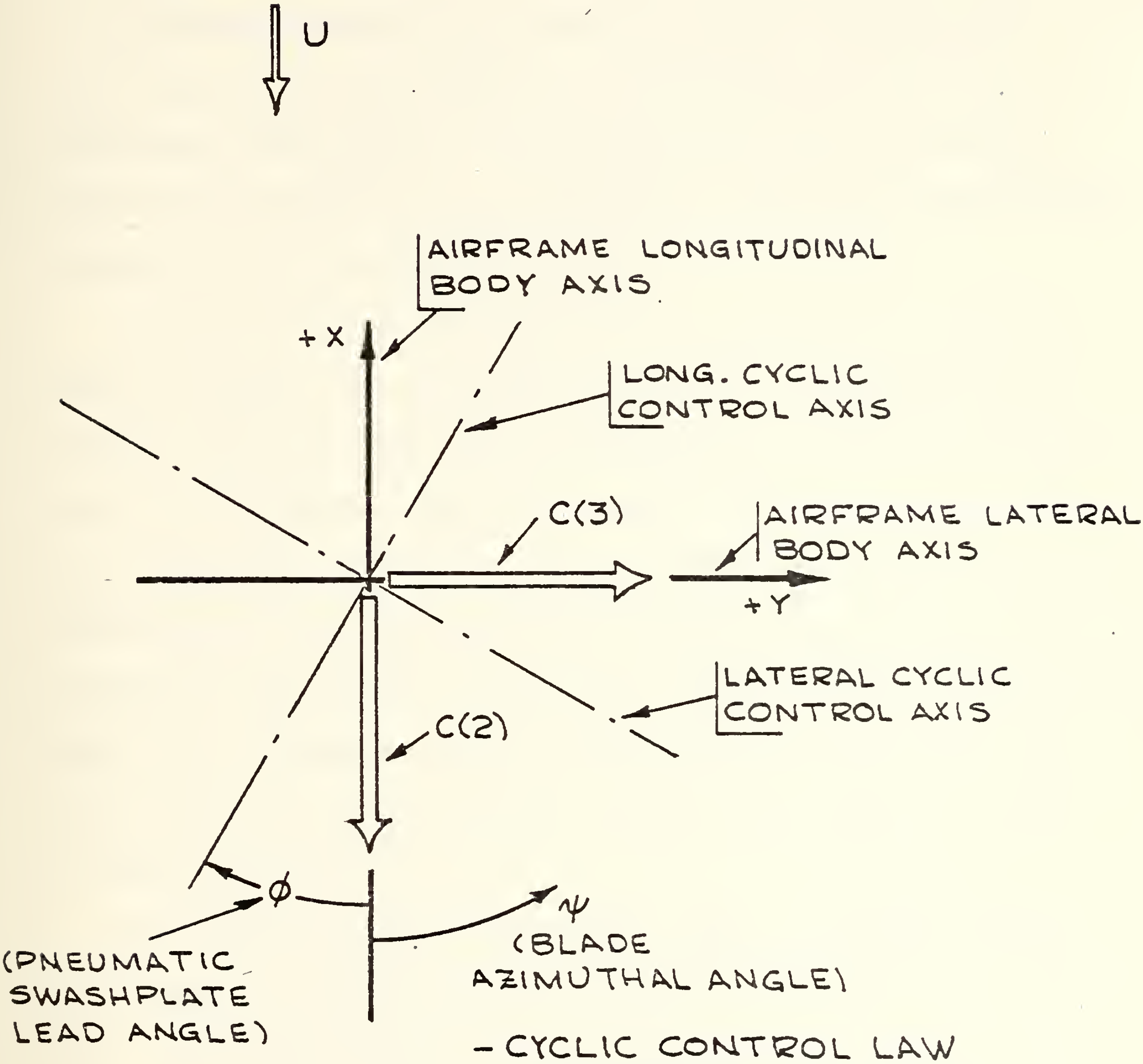
$$B(1) = C(2) \cos \phi - C(3) \sin \phi$$

$$B(2) = C(2) \sin \phi + C(3) \cos \phi$$

where B(1) and B(2) are the longitudinal and lateral cyclic control matrices respectively. No attempt was made to relate these control matrices to actual control stick motions, although reasonable estimates could have been made.

The criteria employed in selecting the control lead angle was the following:

FIGURE 3. VISUALIZATION OF THE PLENUM PRESSURE LEAD ANGLE



– CYCLIC CONTROL LAW TRANSFORMATION –

$$\begin{Bmatrix} B(1) \\ B(2) \end{Bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{Bmatrix} C(2) \\ C(3) \end{Bmatrix}$$

- o Longitudinal cyclic .. produced negligible rolling moment.
- o Lateral cyclic .. produced negligible pitching moment.

Although absolute satisfaction of these constraints concurrently with one choice of lead angle, ϕ, was not physically realizable, it was remarkable that a single value of lead angle, $\phi = 40$ degrees, provided a satisfactory solution in an engineering sense.

Table III lists the variation of pneumatic lead angle versus airspeed for satisfaction of the longitudinal and lateral cyclic constraints respectively. The selection of forty (40) degrees as an engineering answer is in accord with independent analysis done by Kaman Aerospace. The tabulation of the B(1) and B(2) control matrices for a pneumatic lead angle of forty (40) degrees are presented in Table IV. Inspection of the fifth row of B(1) and third row of B(2) provides an indication of the reasonableness of the solution.

The effects of plus and minus five (5) degree changes in pneumatic lead angle will be described during the analysis. This type of sensitivity analysis will provide an indication of the airframe stability root sensitivity in the compensated mode.

C. THE FEEDBACK LAW

The study of the impact of various feedback control laws for reducing or removing the longitudinal instability

TABLE III

Calculations of the Pneumatic Lead Angle vs. Airspeed

$$B(1) = C(2)\cos\phi - C(3)\sin\phi$$

$$B1 = \arctan \frac{C(2)_L}{C(3)_L}$$

$$B(2) = C(2)\sin\phi - C(3)\cos\phi$$

$$\phi_{B2} = \arctan \frac{-C(3)_N}{C(2)_N}$$

HOVER	ϕ_{B1}	= 42.32°
	ϕ_{B2}	= 43°50°
35 KNOTS	ϕ_{B1}	= 37.16°
	ϕ_{B2}	= 39.36°
72 KNOTS	ϕ_{B1}	= 40.87°
	ϕ_{B2}	= 33.09°
110 KNOTS	ϕ_{B1}	= 39.83°
	ϕ_{B2}	= 36.63°
130 KNOTS	ϕ_{B1}	= 40.61°
	ϕ_{B2}	= 42.56°

TABLE IV

Coefficients of the Control Matrix B_{1x} and B_{2x} for pneumodynamic lead angle of (40) degrees

	HOVER	35 KTS	72 KTS	110 KTS	130 KTS
B_{11}	-12.96	-11.13	-8.474	-7.593	-5.121
B_{21}	-8.697	-13.08	-9.193	-22.13	-27.61
B_{31}	9.356	7.565	6.911	7.223	6.726
B_{41}	0.0	0.0	0.0	0.0	0.0
B_{51}	1.164	-1.124	0.295	-0.053	0.174
B_{61}	-0.267	-0.055	-1.446	-0.905	-1.098
B_{71}	4.897	2.816	2.842	3.049	2.030
B_{81}	0.0	0.0	0.0	0.0	0.0
B_{12}	5.486	3.860	2.086	0.639	1.711
B_{22}	-0.765	5.042	2.296	11.21	11.53
B_{32}	-0.572	0.084	0.837	0.425	0.304
B_{42}	0.0	0.0	0.0	0.0	0.0
B_{52}	28.72	22.68	19.52	18.30	16.52
B_{62}	-0.106	-0.643	0.521	0.965	0.585
B_{72}	12.94	10.36	9.244	10.60	9.996
B_{82}	0.0	0.0	0.0	0.0	0.0

began with an attempt to vary the available longitudinal feedback gains and to investigate the eigenvalues and eigenvectors as these gains were varied.

The modified plant matrix, A', was developed in the traditional matrix manner in the uncoupled and fully-coupled state variable format where:

$$A' = A - Bk$$

When only longitudinal cyclic control is considered, the control effectiveness matrix [B] becomes an eight-by-one matrix while the feedback gain coefficient matrix [k] becomes a one-by-eight matrix. The matrix product, B k , is an eight-by-eight matrix.

$$B^T = [U , W , Q , \theta , P , R , V , \phi]$$

$$k = [k_u , k_w , k_q , k_\theta , k_p , k_r , k_v , k_\phi]$$

The earlier confirmation that the unstable root was primarily associated with the longitudinal airframe modes was the reason for only employing feedback in the longitudinal cyclic control when developing the modified plant matrix, [A']. An arbitrary set of moderately damped oscillatory stable roots were selected:

$$\lambda_{1,2} = 1.68 \pm i\ 8.21$$

which corresponded to:

$$\omega_n = \text{undamped natural frequency} = 8.38 \text{ sec}^{-1}$$

$$\zeta = \text{dimensionless damping ratio} = 0.2$$

Since it had been established in the basic plant matrix that the short period mode was the dominant instability, the arbitrarily selected second-order system (a form of modal control) was applied to the augmented two-by-two matrix, A', and the closed form solution calculated to yield values of k_w and k_q. Gain values of $k_w = 0.05$ and $k_q = 0.22$ were determined to yield the desired results but the application of these gains alone, to the uncoupled four-by-four matrix and the fully-coupled eight-by-eight state variable problem failed to produce favorable results and the instability remained with the aircraft matrix at all of the calculated speeds.

The search for the acceptable feedback law using gains of k_w and k_q continued using the HP-9830 with a further modified BASMAT program that would automatically search for acceptable feedback gains in the range of k_w and k_q equal to minus one (-1) to plus one (+1.0). Although values of k_w and k_q could be found that would drive the augmented matrix stable (negative real parts of the eigenvalues) at each calculated speed, these values were not sequentially related to speed and, furthermore, were random in nature, often changing signs more than once as speed increased from hover.

The decision was then made to employ pitch attitude and pitch rate (k_θ and k_q) feedback respectively based on the knowledge that both pitch and pitch rate information were presently available in the H-2 aircraft. Programs existed for both the IBM-360 and the HP-9830 for accomplishing this search for acceptable feedback gains of k_q and k_θ, but while the IBM-360 was much faster in the actual computations (approximately twenty seconds of CPU time were required for the computations of the eigenvalues and eigenvectors for one set of plant, control and feedback gain coefficients vice twenty-five minutes for the HP-9830), the HP-9830 allowed for a much more convenient search. The HP-9830 allowed the programmer to make "in-line" decisions on changes in the feedback gains based on the previous results with approximately thirty minutes between output results. This removed the problem of the long delays encountered because of the turnaround time of the IBM-360. Typical turnaround times were two to five hours depending upon the computer usage at the time of program input. This long turnaround time was a result of the low job priority assigned the program by the computer center, a result of the complexity of the program and the large amount of core memory required.

The rationale of feeding back k_q and k_θ proved fruitful for values of:

$$k_q = 0.45 \quad \text{and} \quad k_\theta = 0.85$$

or the feedback matrix taking the form of:

$$k = [0, 0, 0.45, 0.85, 0, 0, 0, 0]$$

producing stable eigenvalues at all speeds calculated with the exception of hover. These gains did leave an unstable oscillatory root in hover, with a time to double amplitude of 18.15 seconds, over six times the three second minimum time to double amplitude allowed.

D. PNEUMATIC LEAD ANGLE SENSITIVITY CHECK

Concern had been shown over the possibility that variations in the pneumatic lead angle (ϕ) blowing could cause dramatic changes in the stability characteristics of the aircraft. It was recognized that changes in pneumatic lead angle would result in direct changes to the control matrix, B. In order to study the sensitivity to changes in lead angle, new values of the control matrix were computed for pneumatic lead angles of thirty-five (35) and forty-five (45) degrees. The results of these calculations are listed in Table VI. The IBM-360 program was further modified to compute the new eigenvalues and eigenvectors using the basic plant matrix, A, the computed feedback gains, k, given above, and allowing the longitudinal pneumatic control matrix, B, to vary for values of ϕ equal to thirty-five and forty-five degrees. The computed results

TABLE V

Eigenvalues of the augmented matrix, A', at velocities of: Hover, 35, 72, 110 and 130 Knots

Hover:

$$
\begin{aligned}
\lambda &= -0.00019 \\
&= -0.24473 \\
&= -0.47775 \\
&= -0.97459 \\
&= 0.03819 \pm i\,0.27676 \\
&= -6.78748 \pm i\,3.60274
\end{aligned}
$$

35 Knots:

$$
\begin{aligned}
&= -0.01709 \\
&= -0.04164 \\
&= -0.24762 \pm i\,0.81412 \\
&= -0.67857 \pm i\,0.13139 \\
&= -5.67357 \pm i\,3.42617
\end{aligned}
$$

72 Knots:

$$
\begin{aligned}
&= -0.01445 \\
&= -0.07094 \\
&= -0.63074 \pm i\,0.37790 \\
&= -0.43457 \pm i\,1.47940 \\
&= -5.45309 \pm i\,3.07803
\end{aligned}
$$

110 Knots:

$$
\begin{aligned}
&= -0.00326 \\
&= -0.04764 \\
&= -0.49037 \pm i\,0.55734 \\
&= -0.54836 \pm i\,2.06899 \\
&= -5.61868 \pm i\,3.46986
\end{aligned}
$$

130 Knots:

$$
\begin{aligned}
&= -0.01401 \\
&= -0.07035 \\
&= -0.39270 \pm i\,0.61107 \\
&= -0.59046 \pm i\,2.34511 \\
&= -5.31913 \pm i\,3.22033
\end{aligned}
$$

TABLE VI

Variations in the control matrix coefficients with variations in the pneumatic lead angle

	B_{11}	B_{21}	B_{31}	B_{41}	B_{51}	B_{61}	B_{71}	B_{81}
HOVER								
35°	-12.428	-8.730	9.730	0.0	3.662	-0.275	6.006	0.0
40°	-12.955	-8.697	9.356	0.0	1.164	-0.267	4.897	0.0
45°	-13.384	-8.597	9.370	0.0	-1.344	-0.256	3.75	0.0
35 KTS								
35°	-10.750	-9.950	7.543	0.0	0.857	-0.635	3.709	0.0
40°	-11.129	-6.778	7.565	0.0	-1.124	-0.643	2.816	0.0
45°	-11.423	-10.215	7.529	0.0	-3.097	-0.645	1.903	0.0
72 KTS								
35°	-8.260	-11.597	6.957	0.0	1.995	-1.171	3.637	0.0
40°	-8.474	-12.151	6.911	0.0	0.295	-1.237	2.842	0.0
45°	-8.624	-12.612	6.812	0.0	-1.407	-1.293	2.025	0.0
110 KTS								
35°	-7.508	-21.071	7.232	0.0	1.542	-1.167	3.961	0.0
40°	-7.593	-22.132	7.223	0.0	-0.053	-1.232	3.049	0.0
45°	-7.620	-23.024	7.158	0.0	-1.648	-1.288	2.114	0.0
130 KTS								
35°	-4.953	-26.501	6.733	0.0	1.613	-1.071	2.893	0.0
40°	-5.121	-27.611	6.726	0.0	0.714	-1.125	2.030	0.0
45°	-5.251	-28.511	6.674	0.0	-1.266	-1.170	1.151	0.0

showed that the variation of the pneumatic lead angle by plus or minus five (5) degrees had very little effect on the stability characteristics of the augmented plant matrix.

IV. CONCLUSIONS

A study has been made of the basic stability traits of the Kaman Aerospace Corporation XH-2/CCR helicopter, which is presently being constructed under NAVAIR contract as a technology demonstrator for the Circulation Control Rotor concept.

The airframe was defined by contractor generated stability and control derivatives which were then used to develop eigenvalues and eigenvectors for the system. The plant matrix (which characterizes the airframe) was generated by the MOSTAB program as modified to accommodate the CCR system, and the matrix coefficients represent the output from the program when it was operating in the 18 degree-of-freedom situation, i.e., six airframe degrees-of-freedom plus flapping, torsion and lead/lag degrees-of-freedom for the four blades ($6 + 3 \times 4 = 18$). The uncompensated airframe characteristic roots at hover are in close accord with results obtained by the contractor for the six degree-of-freedom airframe, including a mild non-oscillatory instability ($t_2 = 37.1$ sec.) that has been identified as due to a longitudinal short period type mode.

A feedback law has been identified using pitch attitude and pitch rate feedback into the longitudinal cyclic control that provides reasonable characteristic roots for the airframe. Presumably, further improvements could be obtained

by providing feedback in the lateral cyclic control system. The effect of varying the cyclic control lead angle on the pneumatic swashplate was investigated and found to be slight.

It should be noted that a unique feedback control law is not possible in modal control theory, when multiple control inputs (longitudinal and lateral cyclic) are available. Another way of stating this fact is that it is possible with several control inputs (and feedback laws) to have the same eigenvalues, but with different eigenvectors.

Finally, the characterization of the eigenvectors and identification of the eigenvalues with relevant modes was made possible by using a form of root locus analysis where the prime parameter was the amount of cross coupling. The trajectory of the characteristic roots as cross-coupling was linearly altered provided a physical insight into the history of the various roots.

Future studies are suggested to include estimating the mechanical gearing to the cockpit controls and then obtaining airframe response time histories for selected control inputs such as stick doublet type motions. Time histories can be generated quite readily using principles from control theory combined with calculated relations for system's transition matrices. It is quite possible that time history calculations for the compensated airframe will provide a better guide for selecting the control law. In any event, such studies will enhance satisfaction as to the question of airframe response behavior being reasonable.

APPENDIX A

The basic plant matrix of the aircraft linearized equations of motion in the state variable format

THE COUPLED "A" MATRIX

$$
\begin{Bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \\ \dot{p} \\ \dot{r} \\ \dot{v} \\ \dot{\phi} \end{Bmatrix}
=
\left[\begin{array}{cccc|cccc}
X_u & X_w & X_q & -32.2 & X_p & X_r & X_v & 0.0 \\
Z_u & Z_w & U + Z_q & 0.0 & Z_p & Z_r & Z_v & 0.0 \\
M_u + Z_u M_w & M_w + m_w Z_w & M_q + M_w (U+Z_q) & 0.0 & M_p & M_r & M_v & 0.0 \\
0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
\hline
L_u & L_w & L_q & 0.0 & L_p & L_r & L_v & 0.0 \\
N_u & N_w & N_q & 0.0 & N_p & N_r & N_v & 0.0 \\
Y_u & Y_w & Y_q & 0.0 & Y_p & Y_r - U & Y_v & 32.2 \\
0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0
\end{array}\right]
\begin{bmatrix} u \\ w \\ q \\ \theta \\ p \\ r \\ v \\ \phi \end{bmatrix}
$$

U = Aircraft velocity in fps.

APPENDIX B

IBM-360 MODIFIED "BASMAT" PROGRAM

```
C     BASIC MATRIX PROGRAM
C     SUBPROGRAMS USED-  CHREQ, SIMEQ, STMST, PROOT, DET,
C                        CHREGA
      DIMENSION A(10,10),EIGR(10),EIGI(10),C(11),AINV(10,10),
     1     NAME(5),A1(10,10),B(10,5),G(5,10)
 2001 FORMAT (5A4,3I2)
 2002 FORMAT (8E10.2)
 2003 FORMAT (1P6E20.7)
 2004 FORMAT (1H0,5X,16HTHE A MATRIX     ,/)
 2005 FORMAT(1H0,5X,32HTHE CHARACTERISTIC POLYNOMIAL -     ,
     *       24HIN ASCENDING POWERS OF S      /)
 2006 FORMAT (1H0   ,5X,31HTHE EIGENVALUES OF THE A MATRIX)
 2007 FORMAT (9X,9HREAL PART,8X,14HIMAGINARY PART,/)
 2008 FORMAT (1H1,5X,20HBASIC MATRIX PROGRAM)
 2009 FORMAT (6X,23HPROBLEM IDENTIFICATION-,5X,5A4)
 2010 FORMAT (1H0,5X,29HTHE DETERMINANT OF THE MATRIX/)
 2011 FORMAT (1H0,5X,25HTHE INVERSE OF THE MATRIX/)
 2012 FORMAT (1H0,45(1H*))
 2013 FORMAT (6I1)
   24 READ (5,2001) (NAME(I),I=1,5),N,M1,M2
      GO TO (3000,3001,7,10),M2
 3000 DO 1 I=1,N
      C(I)=1.0
    1 READ 2002, (A1(I,K),K=1,N)
      DO 4 I=1,N
      DO 4 K=1,N
    4 A(I,K)=A1(I,K)
      READ 2013, IDET,INV,NRM,ICP,IEIG,ISTM
      PRINT 2008
      PRINT 2009, (NAME(I),I=1,5)
      PRINT 2012
      PRINT 2004
      DO 2 I=1,N
    2 PRINT 2020, (A(I,K),K=1,N)
      IF (M1.EQ.0) GO TO 14
      PRINT 2012
      PRINT 2021
 3001 DO 6 I=1,N
      READ 2002, (B(I,K),K=1,M1)
    6 PRINT 2020, (B(I,K),K=1,M1)
    7 PRINT 2012
      PRINT 2022
      DO 8 I=1,M1
      READ 2002, (G(I,K),K=1,N)
    8 PRINT 2020,(G(I,K),K=1,N)
      PRINT 2023
      DO 13 I=1,N
      DO 12 K=1,N
      TEMP=0.
      DO 11 J=1,M1
   11 TEMP=TEMP + B(I,J)*G(J,K)
   12 A(I,K)=A1(I,K)-TEMP
   13 PRINT 2020, (A(I,K),K=1,N)
   14 IF (IDET.NE.0) GO TO 5
      D=DET(A,N)
      PRINT 2010
      PRINT 2003, D
    5 IF (INV.NE.0) GO TO 15
      PRINT 2011
      CALL SIMEQ(A,C,N,AINV,C,IERR)
      IF (IERR.EQ.0) GO TO 15
      DO 20 I=1,N
   20 PRINT 2003, (AINV(I,J),J=1,N)
   15 CALL CHREQ(A,N,C,NRM)
      CALL PROOT(N,C,EIGR,EIGI,+1)
      IF (ICP.NE.0) GO TO 30
      PRINT 2012
      PRINT 2005
      NN=N+1
      PRINT 2003, (C(I),I=1,NN)
   30 IF (IEIG.NE.0) GO TO 35
      PRINT 2012
      PRINT 2006
      PRINT 2007
      DO 3 I=1,N
    3 PRINT 2003, EIGR(I),EIGI(I)
   35 IF (ISTM.NE.0) GO TO 25
      CALL STMST(N,A,EIGR,EIGI,ISTM)
   25 GO TO 24
   10 STOP
 2020 FORMAT (1P8E15.4)
 2021 FORMAT (1H0,5X,20HTHE CONTROL MATRIX B,/)
 2022 FORMAT (1H0,5X,17HTHE GAIN MATRIX G,/)
 2023 FORMAT (1H0,5X,31HTHE MODIFIED PLANT MATRIX, A-BG,/)
      END
```

```
      SUBROUTINE CHREQ(A,N,C,NRM)
C  THIS SUBROUTINE FINDS THE COEFFICIENTS OF THE CHARACTERISTIC
C     POLYNOMIAL USING THE LEVERRIER ALGORITHM
      COMMON ZED(10,10,10)
      DIMENSION A(10,10),C(11),ATEMP(10,10),PROD(10,10)
      DATA ATEMP/100*0.0/
 1000 FORMAT (1H0,5X,31HTHE MATRIX COEFFICIENTS OF THE    ,
     *       33HNUMERATOR OF THE RESOLVENT MATRIX  )
 1001 FORMAT (1H0,   5X,29HTHE MATRIX COEFFICIENT OF S**,I1/)
 1002 FORMAT (1P6E20.7)
 1003 FORMAT (1H0,45(1H*))
      CALL CHREQA(A,N,C)
      DO 65 I=1,N
   65 ATEMP(I,I)=1.0
   70 DO 80 I=1,N
      DO 80 J=1,N
   80 ZED(N,I,J)=ATEMP(I,J)
      IF (NRM.NE.0) GO TO 71
      WRITE (6,1003)
      WRITE (6,1000)
      M=N-1
      WRITE (6,1001) M
      DO 35 I=1,N
   35 WRITE (6,1002) (ATEMP(I,J),J=1,N)
   71 DO 40 I=1,N
      DO 40 J=1,N
   40 ATEMP(I,J)=A(I,J)
      DO 10 I=1,N
      NNN=N-I
      IF (I.EQ.1) GO TO 55
      IF (NRM.NE.0) GO TO 60
      WRITE (6,1001) NNN
      DO 45 J=1,N
   45 WRITE (6,1002) (ATEMP(J,K),K=1,N)
   60 NP=NNN+1
      DO 90 II=1,N
      DO 90 J=1,N
   90 ZED(NP,II,J)=ATEMP(II,J)
      DO 15 J=1,N
      DO 15 K=1,N
      PROD(J,K)=0.0
      DO 15 L=1,N
   15 PROD(J,K)=PROD(J,K)+(A(J,L)*ATEMP(L,K))
      DO 13 J=1,N
      DO 13 K=1,N
   13 ATEMP(J,K)=PROD(J,K)
   55 DO 10 J=1,N
   10 ATEMP(J,J)=ATEMP(J,J)+C(N-I+1)
      RETURN
      END
```

```
      SUBROUTINE CHREQA(A,N,C)
      DIMENSION J(11),C(11),B(10,10),A(10,10),D(300)
      NN=N+1
      DO 20 I=1,NN
   20 C(I)=0.0
      C(NN)=1.0
      DO 14 M=1,N
      K=0
      L=1
      J(1)=1
      GO TO 2
    1 J(L)=J(L)+1
    2 IF(L-M) 3,5,50
    3 MM=M-1
      DO 4 I=L,MM
      II=I+1
    4 J(II)=J(I)+1
    5 DO 10 I=1,M
      DO 10 KK=1,M
      NR=J(I)
      NC=J(KK)
   10 B(I,KK)=A(NR,NC)
      K=K+1
      D(K)=DET(B,M)
      DO 6 I=1,M
      L=M-I+1
      IF(J(L)-(N-M+L)) 1,6,50
    6 CONTINUE
      M1=N-M+1
      DO 14 I=1,K
   14 C(M1)=C(M1)+D(I)*(-1.0)**M
      RETURN
   50 PRINT 2000
 2000 FORMAT (1H0,5X,15HERROR IN CHREQA)
      RETURN
      END
```

```
      SUBROUTINE [illegible]MED (A,XDOT,KC,AINV,X,IERR)
C     THIS SUBROUTINE FINDS THE INVERSE OF THE MATRIX A USING
C     DIAGONALIZATION PROCEDURES.
      DIMENSION A(10,10),B(10,10),XDOT(11),AINV(10,10),X(11)
      N=1
      IERR=1
      DO 1 I=1,KC
      DO 1 J=1,KC
      AINV(I,J)=0
    1 B(I,J)=A(I,J)
      DO 2 I=1,KC
      AINV(I,I)=1
    2 X(I)=XDOT(I)
      DO 3 I=1,KC
      COMP=0
      K=I
    6 IF(ABS (B(K,I))-ABS (COMP))5,5,4
    4 COMP=B(K,I)
      N=K
    5 K=K+1
      IF(K-KC)6,6,7
    7 IF(B(N,I))8,51,8
    8 IF(N-I)51,12,9
    9 DO 10 M=1,KC
      TEMP=B(I,M)
      B(I,M)=B(N,M)
      B(N,M)=TEMP
      TEMP=AINV(I,M)
      AINV(I,M)=AINV(N,M)
   10 AINV(N,M)=TEMP
      TEMP=X(I)
      X(I)=X(N)
      X(N)=TEMP
   12 X(I)=X(I)/B(I,I)
      TEMP = B(I,I)
      DO 13 M=1,KC
      AINV(I,M)= AINV(I,M)/TEMP
   13 B(I,M) = B(I,M)/TEMP
      DO 16 J=1,KC
      IF(J-I)14,16,14
   14 IF(B(J,I))15,16,15
   15 X(J)=X(J)-B(J,I)*X(I)
      TEMP=B(J,I)
      DO 17 M=1,KC
      AINV(J,M)=AINV(J,M)-TEMP*AINV(I,M)
   17 B(J,M)=B(J,M)-TEMP*B(I,M)
   16 CONTINUE
    3 CONTINUE
      RETURN
   51 PRINT 52
   52 FORMAT (  5X,22HTHE MATRIX IS SINGULAR)
      IERR=0
      RETURN
      END
```

```
      SUBROUTINE STMST (N,A,EIGR,EIGI,IKNOW)
C   THIS SUBROUTINE DETERMINES THE STATE TRANSITION MATRIX USING
C       SYLVESTER-S EXPANSION THEOREM.
      COMMON CHI(10,10,10)
      DIMENSION A(10,10),EIGR(10),EIGI(10),SPS(10,10)
      COMPLEX CA(10,10),CA1(10,10),CA2(10,10),TCA(10,10),
     *  DENOM(10),CEIG(10)
 1000 FORMAT (1H0,5X,25HTHE ELEMENTS OF THE STATE          ,
     *         18H TRANSITION MATRIX      )
 1001 FORMAT(1H0,   5X,25HTHE MATRIX COEFFICIENT OF         ,
     *    5H EXP(,1PE13.6,7H)T*COS(,1PE13.5,2H)T/)
 1002  FORMAT(1P8F15.4)
 1003 FORMAT(1H0,   5X,25HTHE MATRIX COEFFICIENT OF         ,
     *    5H EXP(,1PE13.6,7H)T*SIN(,1PE13.6,2H)T/)
 1004 FORMAT(1H0,   5X,25HTHE MATRIX COEFFICIENT OF         ,
     *                     5H EXP(,1PE13.6, 2H)T/)
 1005 FORMAT (1H0,45(1H*))
      IF(IKNOW.NE.0) GO TO 800
      PRINT 1005
  800 DO 10 K=1,N
      CEIG(K)=CMPLX(EIGR(K),EIGI(K))
      DO 10 L=1,N
   10 CA(K,L) = CMPLX(A(K,L),0.0)
      I=1
      IF(IKNOW.NE.0) GO TO 700
      PRINT 1000
  700 DO 15 K=1,N
   15 DENOM(K)=CEIG(I)-CEIG(K)
      DO 500 J=1,N
      IF (J-I) 100,500,200
  100 IF (J-1) 110,110,150
  200 IF (I-1) 300,300,400
  300 IF (J-I-1) 110,110,150
  400 IF (J-I-1) 110,150,150
  110 DO 5 K=1,N
      DO 5 L=1,N
    5 CA1(K,L)=CA(K,L)
      DO 20 K=1,N
      CA1(K,K)=CA(K,K)-CEIG(J)
      DO 20 L=1,N
   20 CA1(K,L)=CA1(K,L)/DENOM(J)
      GO TO 500
  150 DO 40 K=1,N
      DO 40 L=1,N
   40 CA2(K,L)=CA(K,L)
      DO 25 K=1,N
      CA2(K,K)=CA(K,K)-CEIG(J)
      DO 25 L=1,N
   25 CA2(K,L)=CA2(K,L)/DENOM(J)
      DO 30 K=1,N
      DO 30 L=1,N
      TCA(K,L)=(0.0,0.0)
      DO 30 M=1,N
   30 TCA(K,L)=TCA(K,L) + CA1(K,M)*CA2(M,L)
      DO 35 K=1,N
      DO 35 L=1,N
   35 CA1(K,L)=TCA(K,L)
  500 CONTINUE
      IF (AIMAG(CEIG(I))) 45,50,45
   45 IM=I
      I=I+1
      IF(IKNOW.NE.0) GO TO 801
      PRINT 1001, EIGR(I),EIGI(I)
  801 DO 65 K=1,N
      DO 65 L=1,N
   65 SPS(K,L)= REAL(CA1(K,L))*2.0
      DO 66 K=1,N
      DO 66L=1,N
      CHI(IM,K,L)=SPS(K,L)
   66 CONTINUE
      IF(IKNOW.NE.0) GO TO 802
      DO 80 K=1,N
   80 PRINT 1002, (SPS(K,L),L=1,N)
      PRINT 1003, EIGR(I),EIGI(I)
  802 DO 55 K=1,N
      DO 55 L=1,N
   55 SPS(K,L)=AIMAG(CA1(K,L))*2.0
      DO 56 K=1,N
```

```
      DO 56 L=1,N
      CHI(I  ,K,L)=SPS(K,L)
   56 CONTINUE
      IF(IKNOW.NE.0) GO TO 600
      DO 85 K=1,N
   85 PRINT 1002, (SPS(K,L),L=1,N)
C ** CALCULATE COMPLEX EIGENVECTOR (A MODIFICATION)
  210  PRINT 1005
  220  DO 221 K=1,N
       SPS(K,1)=SQRT(CHI(IM,K,1)**2 + CHI(I,K,1)**2)
       SPS(K,2)=ATAN2(CHI(I,K,1),CHI(IM,K,1)) *(-57.2958)
       PRINT 1007,SPS(K,1),SPS(K,2)
  221  CONTINUE
 1005  FORMAT(1H0,5X,26HTHE COMPLEX EIGENVECTOR IS,/8X,9HMAGNITUDE,
     1  10X,11HPHASE (DEG)/)
 1007  FORMAT(1P2E20.7)
C ** END OF MODIFICATION.
      GO TO 600
   50 IF(IKNOW.NE.0) GO TO 804
      PRINT 1004, EIGR(I)
  804 DO 60 K=1,N
      DO 60 L=1,N
   60 SPS(K,L)=REAL (CA1(K,L))
      DO 61 K=1,N
      DO 61 L=1,N
      CHI(I,K,L)=SPS(K,L)
   61 CONTINUE
      IF(IKNOW.NE.0) GO TO 600
      DO 75 K=1,N
   75 PRINT 1002, (SPS(K,L),L=1,N)
  600 IF (I.GE.N) RETURN
      I=I+1
      GO TO 700
      END
```

```
      SUBROUTINE BROOT(N,A,U,V,IR)
C  THIS SUBROUTINE USES A MODIFIED BARSTOW METHOD TO FIND THE
C     ROOTS OF A POLYNOMIAL.
      DIMENSION A(20),U(20),V(20),H(21),B(21),C(21)
      IREV=IR
      NC=N+1
      DO1I=1,NC
    1 H(I)=A(I)
      P=0.
      Q=0.
      R=0.
    3 IF(H(1))4,2,4
    2 NC=NC-1
      V(NC)=0.
      U(NC)=0.
      DO1002I=1,NC
 1002 H(I)=H(I+1)
      GOTO3
    4 IF(NC-1)5,100,5
    5 IF(NC-2)7,6,7
    6 R=-H(1)/H(2)
      GOTO50
    7 IF(NC-3)9,8,9
    8 P=H(2)/H(3)
      Q=H(1)/H(3)
      GOTO70
    9 IF(ABS (H(NC-1)/H(NC))-ABS (H(2)/H(1)))10,19,19
   10 IREV=-IREV
      M=NC/2
      DO11I=1,M
      NL=NC+1-I
      F=H(NL)
      H(NL)=H(I)
   11 H(I)=F
      IF(Q)13,12,13
   12 P=0.
      GOTO15
   13 P=P/Q
      Q=1./Q
   15 IF(R)16,19,16
   16 R=1./R
   19 E=5.E-10
      B(NC)=H(NC)
      C(NC)=H(NC)
      B(NC+1)=0.
      C(NC+1)=0.
      NP=NC-1
   20 DO49J=1,1000
      DO21I1=1,NP
      I=NC-I1
      B(I)=H(I)+R*B(I+1)
   21 C(I)=B(I)+R*C(I+1)
      IF(ABS (B(1)/H(1))-E)50,50,24
   24 IF(C(2))23,22,23
   22 R=R+1.
      GOTO30
```

```
   23 R=R-B(1)/C(2)
   30 DO37II=1,NP
      I=NC-II
      B(I)=H(I)-P*B(I+1)-Q*B(I+2)
   37 C(I)=B(I)-P*C(I+1)-Q*C(I+2)
      IF(H(2))32,31,32
   31 IF(ABS (B(2)/H(1))-E)33,33,34
   32 IF(ABS (B(2)/H(2))-E)33,33,34
   33 IF(ABS (B(1)/H(1))-E)70,70,34
   34 CBAR=C(2)-B(2)
      D=C(3)**2-CBAR*C(4)
      IF(D)36,35,36
   35 P=P-2.
      Q=Q*(Q+1.)
      GOTO49
   36 P=P+(B(2)*C(3)-B(1)*C(4))/D
      Q=Q+(-B(2)*CBAR+B(1)*C(3))/D
   49 CONTINUE
      E=E*10.
      GOTO20
   50 NC=NC-1
      V(NC)=0.
      IF(IREV)51,52,52
   51 U(NC)=1./R
      GOTO53
   52 U(NC)=R
   53 DO54I=1,NC
   54 H(I)=B(I+1)
      GOTO4
   70 NC=NC-2
      IF(IREV)71,72,72
   71 QP=1./Q
      PP=P/(Q*2.0)
      GOTO73
   72 QP=Q
      PP=P/2.0
   73 F=(PP)**2-QP
      IF(F)74,75,75
   74 U(NC+1)=-PP
      U(NC)=-PP
      V(NC+1)=SQRT (-F)
      V(NC)=-V(NC+1)
      GOTO76
   75 IF(PP)81,80,81
   80 U(NC+1)=-SQRT(F)
      GO TO 82
   81 U(NC+1)=-(PP/ABS (PP))*(ABS (PP)+SQRT (F))
   82 CONTINUE
      V(NC+1)=0.
      U(NC)=QP/U(NC+1)
      V(NC)=0.
   76 DO77I=1,NC
   77 H(I)=B(I+2)
      GOTO4
  100 RETURN
      END
```

APPENDIX C

MODIFIED "BASMAT" PROGRAM FOR HP-9830

```
1 CIM A(E,8),O(8,8)
2 CIM M(8,1),Y(1,8)
4 CIM Z(8,8)
6 FIXEC 6
7 CATA
8 CATA
9 CATA
10 CATA
11 CATA
12 CATA
13 CATA
14 CATA
15 DATA
16 MAT READ O
17 PRINT "CCR 130 KNCTS THE BASIC A MATRIX"
18 MAT PRINT C
19 PRINT
20 MAT READ M
21 PRINT "THE BASIC CONTRCL MATRIX IS"
22 MAT PRINT M
23 FOR I5=-1 TO 1 STEP C.1
24 FOR J5=-1 TO 1 STEP C.1
25 Y(1,1)=Y(1,2)=Y(1,5)=Y(1,6)=Y(1,7)=Y(1,8)=0
27 Y(1,3)=I5
28 Y(1,4)=J5
29 MAT Z=M*Y
30 MAT A=C-Z
31 G1=C
32 GCSUB 2C00
33 N=8
34 PRINT"THE REVISEC PLANT MATRIX AT 130 KTS, WITH K(Q)=",I5
35 PRINT"ANC K(THETA)=",J5
36 MAT PRINT A
37 REDIM K(8,8)
38 PRINT
43 FOR I=1 TO N
44 F(I,1)=M(I,1)
46 NEXT I
48 FCR I=2 TO N
50 L5=I-1
52 FCR J=1 TC N
54 F(J,I)=0
56 FOR K=1 TC N
58 F(J,I)=F(J,I)+A(J,K)*F(K,L5)
6C NEXT K
61 NEXT J
62 NEXT I
63 FCR I=1 TC N
64 FCR J=1 TO N
65 K(I,J)=A(I,J)
66 A(I,J)=F(I,J)
67 NEXT J
68 NEXT I
69 GCSUB 1C00
70 IF E NCT ECUAL TC ZERO THEN 76
71 GCSUB 2C00
72 PRINT "MATRIX IS UNCCNTRCLLABLE"
74 GCTC 110
76 FCR I=1 TC N
78 FCR J=1 TO N
79 Q(I,J)=C
80 FCR K=1 TO N
81 Q(I,J)=Q(I,J)+F(I,K)*P(K,J)
28 NEXT K
82 NEXT K
84 NEXT J
86 NEXT I
88 E1=C
90 FCR I=1 TO N
92 FCR J=1 TO N
93 A(I,J)=K(I,J)
94 IF (I-J) NCT ECUAL TC ZERC THEN 100
96 E2=ABS(C(I,J)-1)
98 GOTC 101
10C E2=ABS(C(I,J))
101 IF ABS(E1)>ABS(E2) THEN 104
102 E1=ABS(E2)
103 GCTC 1C5
104 E1=ABS(E1)
105 NEXT J
10E NEXT I
107 PRINT
10E IF (E1-1E-C5)<O THEN 11C
109 PRINT "PLAINT IS NUMERICALLY UNCONTROLLABLE, DEVIATICN=";E1
11C GCSUB 2CCC
112 PRINT "OPEN LCCP CALCULATICNS"
114 PRINT
116 PRINT "GENEM COEF IN ASCENDING POWERS OF S"
```

```
120 GOSUB 3199
122 FOR I=1 TO N6
124 PRINT C(I)
126 L(I)=C(I)
128 NEXT I
130 GOSUB 3399
132 GOSUB 5000
430 NEXT J5
440 NEXT I5
450 END
1000 N7=1
1002 E=1
1004 FOR I=1 TO N
1006 FOR J=1 TO N
1008 P(I,J)=0
1010 G(I,J)=A(I,J)
1012 NEXT J
1014 NEXT I
1016 FOR I=1 TO N
1018 X(I)=1
1019 P(I,I)=1
1020 NEXT I
1022 FOR I=1 TO N
1024 C=0
1026 K=I
1028 IF (ABS(G(K,I))-ABS(C)) <= 0 THEN 1034
1030 C=G(K,I)
1032 N7=K
1034 K=K+1
1036 IF (K-N) <= 0 THEN 1028
1038 IF (G(N7,I))=0 THEN 1102
1040 IF (N7-I)<0 THEN 1102
1042 IF (N7-I)=0 THEN 1066
1044 FOR M=1 TO N
1046 T=G(I,M)
1048 G(I,M)=G(N7,M)
1050 G(N7,M)=T
1052 T=P(I,M)
1054 P(I,M)=P(N7,M)
1056 P(N7,M)=T
1058 NEXT M
1060 T=X(I)
1062 X(I)=X(N7)
1064 X(N7)=T
1066 X(I)=X(I)/G(I,I)
1068 T=X(I)
1068 T=G(I,I)
1070 FOR M=1 TO N
1072 P(I,M)=P(I,M)/T
1074 G(I,M)=G(I,M)/T
1076 NEXT M
1078 FOR J=1 TO N
1080 IF (J-I)=0 THEN 1096
1082 IF (G(J,I))=0 THEN 1096
1084 X(J)=X(J)-G(J,I)*X(I)
1086 T=G(J,I)
1088 FOR N7=1 TO N
1090 P(J,N7)=P(J,N7)-T*P(I,N7)
1092 G(J,N7)=G(J,N7)-T*G(I,N7)
1094 NEXT N7
1096 NEXT J
1098 NEXT I
1100 RETURN
1102 PRINT "MATRIX IS SINGULAR"
1104 E=0
1106 RETURN
1500 FOR I=1 TO N6
1502 R(I)=C(I)
1504 NEXT I
1506 RETURN
2000 PRINT "************************************************"
2004 RETURN
3199 N6=N+1
3200 DIM C(100)
3201 FOR I=1 TO N6
3203 C(I)=0
3205 NEXT I
3207 C(N6)=1
3209 FOR M=1 TO N
3211 A=0
3213 L=1
3215 J(1)=1
3217 GOTO 3221
3219 J(L)=J(L)+1
3221 IF (L-M)=0 THEN 3235
3223 IF (L-M)>0 THEN 3277
3225 M1=M-1
3227 FOR I=L TO M1
3229 I1=I+1
```

```
3231 J(I1)=J(I)+1
3233 NEXT I
3235 FOR I=1 TO M
3237 FOR K1=1 TO M
3239 N5=J(I)
3241 N1=J(K1)
3243 B(I,K1)=A(N5,N1)
3245 NEXT K1
3247 NEXT I
3249 A=A+1
3251 GOSUB 3299
3253 C(A)=D
3255 FOR I=1 TO M
3257 L=M-I+1
3259 IF (J(L)-(N-M+L))<C THEN 3219
3261 IF (J(L)-(N-M+L))>0 THEN 3277
3263 NEXT I
3265 M2=N-M+1
3267 FOR I=1 TO A
3269 C(M2)=C(M2)+C(I)*(-1)EXP(M)
3271 NEXT I
3273 NEXT M
3275 RETURN
3277 PRINT " ERROR IN CFRECA"
3279 RETURN
3299 I3=C
3301 FOR I=1 TO M
3303 K=I
3305 IF (B(K,I)) NOT EQUAL TO ZERO THEN 3313
C THEN 3313
3307 K=K+1
3309 IF (K-M) <= 0 THEN 3305
3311 IF (K-M)> C THEN 3359
3313 IF (I-K)= 0 THEN 3329
3315 IF (I-K)> C THEN 3359
3317 FOR M4=1 TO M
3319 T=B(I,M4)
3321 B(I,M4)=B(K,M4)
3323 B(K,M4)=T
3325 NEXT M4
3327 I3=I3+1
3329 I1=I+1
3331 IF I1>M THEN 3345
3333 FOR M4=I1 TO M
3335 IF B(M4,I)=C THEN 3344
3337 T=B(M4,I)/B(I,I)
3339 FOR B=I TO M
3341 B(M4,B)=B(M4,B)-B(I,B)*T
3343 NEXT B
3344 NEXT M4
3347 D=1
3349 FOR I=1 TO M
3351 D=D*B(I,1)
3353 NEXT I
3355 D=((-1)*EXP(I3)*D
3357 RETURN
3359 D=C
3361 RETURN
3399 I3=1
3401 N1=N+1
3403 FOR I=1 TO N1
3405 D(I)=C(I)
3407 NEXT I
3409 P=Q=R=0
3411 IF D(1) NOT EQUAL TO ZERO THEN 3425
0 THEN 3425
3413 N1=N1-1
3415 V(N1)=U(N1)=C
3417 FOR I=1 TO N1
3419 D(I)=D(I+1)
3421 NEXT I
3423 GOTO 3411
3425 IF (N1-1)=C THEN 3615
3427 IF (N1-2) NOT EQUAL TO ZERO THEN 3433
3429 R=-D(1)/D(2)
3431 GOTO 3549
3433 IF (N1-3) NOT EQUAL TO ZERO THEN 3441
3435 P=D(2)/D(3)
3437 Q=D(1)/D(3)
3439 GOTO 3567
3441 IF (ABS(D(N1-10/D(N1)-ABS(D(2)/D(1)) >= 0 THEN 3437
3443 I3=-I3
3445 M=N1/2
3447 FOR I=1 TO M
3449 N2=N1+1-I
3451 F=D(N2)
3453 D(N2)1D(I)
3453 D(N2)=D(I)
```

```
3455 C(I)=F                                                     BAS02660
3457 NEXT I                                                     BAS02670
3459 IF Q NOT EQUAL TO ZERO THEN 3465                           BAS02680
3461 P=0                                                        BAS02690
3463 GO TO 3469                                                 BAS02700
3465 P=P/Q                                                      BAS02710
3467 Q=1/Q                                                      BAS02720
3469 IF R=0 THEN 3473                                           BAS02730
3471 R=1/R                                                      BAS02740
3473 E=5E-10                                                    BAS02750
3475 J(N1)=C(N1)                                                BAS02760
3477 W(N1)=C(N1)                                                BAS02770
3479 J(N1+1)=W(N1+1)=0                                          BAS02780
3483 N3=N1-1                                                    BAS02790
3485 FOR L=1 TO 1000                                            BAS02800
3487 FOR I1=1 TO N3                                             BAS02810
3489 I=N1-I1                                                    BAS02820
3491 J(I)=C(I)+R*J(I+1)                                         BAS02830
3493 W(I)=J(I)+R*W(I+1)                                         BAS02840
3495 NEXT I1                                                    BAS02850
3497 IF (ABS(J(1)/D(1)-E)<= 0 THEN 3549                         BAS02860
3499 IF W(2) NOT EQUAL TO ZERO THEN 3505                        BAS02870
3501 R=R+1                                                      BAS02880
3503 GOTO 3507                                                  BAS02890
3505 R=R-J(1)/W(2)                                              BAS02900
3507 FOR I1=1 TO N3                                             BAS02910
3509 I=N1-I1                                                    BAS02920
3511 J(I)=C(I)-P*J(I+1)-Q*J(I+2)                                BAS02930
3513 W(I)=J(I)-P*W(I+1)-Q*W(I+2)                                BAS02940
3515 NEXT I1                                                    BAS02950
3517 IF C(2) NOT EQUAL TO ZERO THEN 3523                        BAS02960
3519 IF (ABS(J(2)/D(1))_E) > THEN 3527                          BAS02970
3521 IF (ABS(J(2)/D(1))-E) <= THEN 3525                         BAS02980
3523 IF (ABS(J(2)/D(2))-E) > 0 THEN 3527                        BAS02990
3525 IF (ABS(J(1)/D(1))-E ) <= 0 THEN 3567                      BAS03000
3527 C1=W(2)-J(2)                                               BAS03010
3529 D=W(3)*EXP(2)-C1*W(4)                                      BAS03020
3531 IF  D NOT EQUAL TO ZERO THEN 3539                          BAS03030
3533 P=P-2                                                      BAS03040
3535 Q=Q*(Q+1)                                                  BAS03050
3537 GOTO 3543                                                  BAS03060
3539 P=P+(J(2)*W(3)-J(1)*W(4))/D                                BAS03070
3541 Q=Q+(-J(2)*C1+J(1)*W(3))/D                                 BAS03080
3543 NEXT L                                                     BAS03090
3545 E=E*10                                                     BAS03100
3547 GOTO 3485                                                  BAS03110
3549 N1=N1-1                                                    BAS03120
3551 V(N1)=0                                                    BAS03130
3553 IF I3 >= 0 THEN 3559                                       BAS03140
3555 U(N1)=1/R                                                  BAS03150
3557 GOTO 3561                                                  BAS03160
3559 U(N1)=R                                                    BAS03170
3561 FOR I=1 TO N1                                              BAS03180
3563 D(I)=J(I+1)                                                BAS03190
3564 NEXT I                                                     BAS03200
3565 GOTO 3425                                                  BAS03210
3567 N1=N1-2                                                    BAS03220
3569 IF I3 >= 0 THEN 3577                                       BAS03230
3571 Q1=1/Q                                                     BAS03240
3573 P1=P/(Q*2)                                                 BAS03250
3575 GOTO 3581                                                  BAS03260
3577 Q1=Q                                                       BAS03270
3579 P1=P/2                                                     BAS03280
3581 F=P1*EXP(2)-Q1                                             BAS03290
3583 IF F >= 0 THEN 3593                                        BAS03300
3585 U(N1+1)=U(N1)=-P1                                          BAS03310
3587 V(N1+1)=SQR(-F)                                            BAS03320
3589 V(N1)=-V(N1+1)                                             BAS03330
3591 GOTO 3607                                                  BAS03340
3593 IF P1 NOT EQUAL TO ZERO THEN 3599                          BAS03350
3595 U(N1+1)=-SQR(F)                                            BAS03360
3597 GOTO 3601                                                  BAS03370
3599 U(N1+1)=-(P1/ABS(P1))*(ABS(P1)+SQR(F))                     BAS03380
3601 V(N1+1)=V(N1)=0                                            BAS03390
3603 U(N1)=Q1/U(N1+1)                                           BAS03400
3607 FOR I=1 TO N1                                              BAS03410
3609 C(I)=J(I+2)                                                BAS03420
3611 NEXT I                                                     BAS03430
3613 GOTO 3425                                                  BAS03440
3615 RETURN                                                     BAS03450
5000 RETURN                                                     BAS03460
5001 PRINT                                                      BAS03470
5002 PRINT " THE ROOTS ARE:              REAL          INAGINARY" BAS03480
5004 FOR I=1 TO N                                               BAS03490
5006 PRINT "                  ";U(I),V(I))                      BAS03500
5008 NEXT I                                                     BAS03510
5010 PRINT                                                      BAS03520
5012 RETURN                                                     BAS03530
```

APPENDIX D

Sample BASMAT computer program output.

```
BASIC MATRIX PROGRAM
PROBLEM IDENTIFICATION-     HOVER 40 DEG 1.0
*********************************************

THE A MATRIX

-1.7090D-03   -1.4430D-03    2.3050D 00   -3.2200D 01    7.6930D-01   -2.7350D-01    2.2310D-03    0.0
-5.0330D-03   -2.3660D-01   -1.4170D-01    0.0           6.7200D-01    6.5740D-01   -1.7380D-02    0.0
 8.2927D-04   -3.1980D-04   -2.6420D 00    0.0          -1.9810D 00    1.6880D-01   -1.0340D-03    0.0
 0.0           0.0           1.0000D 00    0.0           0.0           0.0           0.0           0.0
-4.3990D-03   -1.7060D-03    6.4590D 00    0.0          -7.7130D 00   -2.7480D-01   -1.3190D-02    0.0
 3.5200D-05   -1.9000D-03    1.3150D-01    0.0           2.0450D-01   -3.6610D-01    1.2570D-02    0.0
-2.0400D-03   -1.4240D-02    7.2510D-01    0.0          -2.4770D 00    5.6080D-01   -2.6050D-02    3.2200D 01
 0.0           0.0           0.0           0.0           1.0000D 00    0.0           0.0           0.0
*********************************************

THE CONTROL MATRIX B

-1.2955D 01
-8.6970D 00
 9.3560D 00
 0.0
 1.1640D 00
-2.6700D-01
 4.8970D 00
 0.0
*********************************************

THE GAIN MATRIX G

 0.0           0.0           0.0           0.0           0.0           0.0           0.0           0.0

THE MODIFIED PLANT MATRIX, A-BG

-1.7090D-03   -1.4430D-03    2.3050D 00   -3.2200D 01    7.6930D-01   -2.7350D-01    2.2310D-03    0.0
-5.0330D-03   -2.3660D-01   -1.4170D-01    0.0           6.7200D-01    6.5740D-01   -1.7380D-02    0.0
 8.2927D-04   -3.1980D-04   -2.6420D 00    0.0          -1.9810D 00    1.6880D-01   -1.0340D-03    0.0
 0.0           0.0           1.0000D 00    0.0           0.0           0.0           0.0           0.0
-4.3990D-03   -1.7060D-03    6.4590D 00    0.0          -7.7130D 00   -2.7480D-01   -1.3190D-02    0.0
 3.5200D-05   -1.9000D-03    1.3150D-01    0.0           2.0450D-01   -3.6610D-01    1.2570D-02    0.0
-2.0400D-03   -1.4240D-02    7.2510D-01    0.0          -2.4770D 00    5.6080D-01   -2.6050D-02    3.2200D 01
 0.0           0.0           0.0           0.0           1.0000D 00    0.0           0.0           0.0
*********************************************

THE CHARACTERISTIC POLYNOMIAL - IN ASCENDING POWERS OF S

 -1.1271783D-04      3.3005423D-03      1.4150299D-01      1.0259979D 00      5.2725677D 00      2.1905121D 01
  3.9805609D 01      1.0565645D 01      1.0000000D 00
```

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THE EIGENVALUES OF THE A MATRIX

REAL PART	IMAGINARY PART
1.8669760D-02	0.0
-5.1895065D-02	0.0
-6.3446466D-03	-2.0903866D-01
-6.3446466D-03	2.0903866D-01
-2.8205846D-01	-1.6011221D-02
-2.8205846D-01	1.6011221D-02
-5.1878087D 00	-2.5325661D 00
-5.1878087D 00	2.5325661D 00

THE ELEMENTS OF THE STATE TRANSITION MATRIX

THE MATRIX COEFFICIENT OF EXP(1.866976D-02T)

1.8721D-01	1.1740D 00	-2.4165D 02	-3.2289D 02	-4.8103D 01	-7.0493D 01	-4.8606D-01	-8.3832D 02
-4.1985D-03	-2.6323D-02	5.4192D 00	7.2412D 00	1.0788D 00	1.5809D 00	1.0901D-02	1.8800D 01
-2.1214D-06	-1.2966D-05	2.7667D-03	3.5710D-03	4.8970D-04	7.7467D-04	5.2796D-06	9.2726D-03
-1.1090D-04	-6.9552D-04	1.4314D-01	1.9129D-01	2.8506D-02	4.1762D-02	2.8797D-04	4.9663D-01
-4.6798D-07	-2.5265D-06	6.3754D-04	6.9414D-04	3.9662D-05	1.4358D-04	9.7266D-07	1.8042D-03
-1.1963D-03	-7.5021D-03	1.5441D 00	2.0633D 00	3.0728D-01	4.5045D-01	3.1059D-03	5.3568D 00
-3.7749D-02	-2.3673D-01	4.8726D 01	6.5107D 01	9.6993D 00	1.4214D 01	9.8008D-02	1.6904D 02
-2.1557D-05	-1.3528D-04	2.7815D-02	3.7206D-02	5.5521D-03	3.1244D-03	5.6032D-05	9.6557D-02

THE MATRIX COEFFICIENT OF EXP(-5.189506D-02T)

4.8772D-01	-1.7925D 00	2.7144D 02	3.0262D 02	6.0207D 01	8.6182D 01	-1.5766D 00	9.7825D 02
-1.7473D-02	6.4234D-02	-9.7249D 00	-1.0842D 01	-2.1570D 00	-3.0876D 00	5.6484D-02	-3.5047D 01
-4.0543D-05	1.4938D-04	-2.2773D-02	-2.5210D-02	-4.9432D-03	-7.1710D-03	1.3151D-04	-8.1459D-02
7.8291D-04	-2.8762D-03	4.3577D-01	4.8579D-01	9.6634D-02	1.3835D-01	-2.5309D-03	1.5704D 00
-8.5392D-06	3.1821D-05	-5.0505D-03	-5.3670D-03	-1.0435D-03	-1.5149D-03	2.8091D-05	-1.7353D-02
-3.7493D-03	1.3763D-02	-2.0867D 00	-2.3264D 00	-4.6284D-01	-6.6251D-01	1.2120D-02	-7.5201D 00
-9.7159D-02	3.5717D-01	-5.4075D 01	-6.0285D 01	-1.1994D 01	-1.7168D 01	3.1408D-01	-1.9488D 02
1.6668D-04	-6.1293D-04	9.2830D-02	1.0344D-01	2.0563D-02	2.9454D-02	-5.3892D-04	3.3437D-01

THE MATRIX COEFFICIENT OF EXP(-6.344647D-03T)*COS(2.090386D-01T)

3.2446D-01	1.1425D-01	-1.7191D 01	1.9283D 01	-9.2293D 00	1.1777D 00	1.5591D 00	-9.7343D 01
-5.4734D-03	-1.5052D-03	1.9924D-01	-1.9633D-01	9.8316D-02	-1.9689D-02	-2.6371D-02	1.0189D 00
4.4761D-04	1.4439D-04	-2.0850D-02	2.2562D-02	-1.0945D-02	1.6179D-03	2.1531D-03	-1.1477D-01
-7.0069D-04	9.4587D-04	-2.1949D-01	3.2300D-01	-1.4230D-01	-2.0425D-03	-3.5642D-03	-1.5519D 00
-1.9237D-04	-8.1428D-05	-2.5868D-03	6.7736D-03	-2.6321D-03	-6.7515D-04	-9.3416D-04	-3.0267D-02
4.1326D-03	1.3870D-03	-2.0429D-01	2.2476D-01	-1.0827D-01	1.4971D-02	1.5869D-02	-1.1391D 00
1.3719D-01	6.9875D-03	1.5152D 00	-4.4704D 00	1.7142D 00	4.8063D-01	6.6607D-01	1.9844D 01
-2.1036D-04	-5.0313D-04	1.0333D-01	-1.4358D-01	6.4304D-02	-3.4308D-04	-9.3997D-04	6.9651D-01

THE MATRIX COEFFICIENT OF EXP(-6.344647D-03T)*SIN(2.090396D-01T)

1.1545D-01	-1.4346D-01	3.4720D 01	-4.9393D 01	2.1745D 01	3.4182D-01	5.8520D-01	2.3721D 02
-1.1026D-03	2.5654D-03	-5.7676D-01	8.3716D-01	-3.7025D-01	-2.7878D-03	-5.8204D-03	-4.0312D 00
1.3302D-04	-2.0250D-05	4.6551D-02	-6.8266D-02	3.0110D-02	3.7860D-04	6.6040D-04	3.2818D-01
2.1201D-03	7.1650D-04	-1.0651D-01	1.1774D-01	-5.6631D-02	7.6836D-03	1.0192D-02	-5.9613D-01
4.9857D-05	1.0552D-04	-2.1561D-02	2.9835D-02	-1.3374D-02	2.1837D-04	2.2503D-04	-1.4481D-01
1.3350D-03	-1.8510D-03	4.2829D-01	-6.2975D-01	2.7752D-01	3.8721D-03	6.7985D-03	3.0261D 00
-3.3212D-02	-7.4854D-02	1.5339D 01	-2.1269D 01	9.5295D 00	-1.4703D-01	-1.4516D-01	1.0320D 02
-9.2655D-04	-5.4212D-05	-9.3967D-03	2.8046D-02	-1.0624D-02	-3.2490D-03	-4.4973D-03	-1.2365D-01

THE COMPLEX EIGENVECTOR IS

MAGNITUDE	PHASE (DEG)
3.4438412D-01	-1.9587305D 01
5.5847385D-03	1.6853984D 02
4.6695805D-04	-1.6551415D 01
2.2328593D-03	-1.0828504D 02
1.0872241D-04	-1.6546956D 02
4.3428603D-03	-1.7902748D 01
1.4115556D-01	1.3608428D 01
9.5013255D-04	1.0279140D 02

THE MATRIX COEFFICIENT OF EXP(-2.820585D-01)T*COS(1.601122D-02)T

-4.4127D-04	5.0438D-01	-1.0566D 01	9.7992D-01	-3.5463D 00	-1.6978D 01	5.0182D-01	-4.2576D 01
2.7120D-02	9.6361D-01	4.1896D 00	3.7965D 00	1.0465D 00	1.5203D 00	-4.0952D-02	1.5228D 01
4.7320D-05	-2.2030D-04	4.5755D-02	2.8162D-03	1.4716D-02	6.4665D-02	-1.9047D-03	1.8141D-01
-8.7458D-05	2.6024D-03	-1.2976D-01	-8.0453D-04	-4.2115D-02	-1.9102D-01	5.6338D-03	-5.1378D-01
8.9843D-06	-1.9568D-04	1.1870D-02	2.2811D-04	3.8418D-03	1.7320D-02	-5.1080D-04	4.7051D-02
7.7574D-04	-7.6719D-03	8.3095D-01	3.8313D-02	2.6901D-01	1.1919D 00	-3.5120D-02	3.3025D 00
-1.2667D-03	-1.2730D-01	1.4653D 00	-3.4218D-01	5.0945D-01	2.6113D 00	-7.7378D-02	5.9854D 00
-7.0943D-06	1.2535D-03	-2.1492D-02	2.0231D-03	-1.0484D-02	-4.5465D-02	1.4612D-03	-1.2445D-01

THE MATRIX COEFFICIENT OF EXP(-2.820585D-01)T*SIN(1.601122D-02)T

1.5951D-01	3.5831D 00	6.6710D 01	1.8154D 01	2.0299D 01	7.6883D 01	-2.2452D 00	2.5919D 02
1.0998D-01	2.3822D 00	4.7782D 01	1.2340D 01	1.4596D 01	5.5894D 01	-1.6361D 00	1.8591D 02
-3.9816D-04	-9.0964D-03	-1.6323D-01	-4.5600D-02	-4.9745D-02	-1.8701D-01	5.4688D-03	-6.3465D-01
1.4162D-03	3.2081D-02	5.8680D-01	1.6171D-01	1.7841D-01	6.7383D-01	-1.9710D-02	2.2792D 00
-1.2362D-04	-2.8013D-03	-5.0731D-02	-1.4108D-02	-1.5560D-02	-5.8616D-02	1.7138D-03	-1.9834D-01
-7.7812D-03	-1.7722D-01	-3.2052D 00	-8.9047D-01	-9.7370D-01	-3.6713D 00	1.0738D-01	-1.2446D 01
-3.0559D-02	-6.8435D-01	-1.2909D 01	-3.4805D 00	-3.9313D 00	-1.4919D 01	4.3651D-01	-5.0172D 01
4.3815D-04	9.8604D-03	1.8277D-01	4.9900D-02	5.5647D-02	2.1056D-01	-6.1595D-03	7.1036D-01

THE COMPLEX EIGENVECTOR IS

MAGNITUDE	PHASE (DEG)
1.5951015D-01	-9.0159253D 01
1.1327752D-01	-7.6148246D 01
4.0095997D-04	8.3222236D 01
1.4188623D-03	-9.3533577D 01
1.2394608D-04	8.5843212D 01
7.8197876D-03	8.4306825D 01
3.0684700D-02	9.2366032D 01
4.3820371D-04	-9.0926353D 01

THE MATRIX COEFFICIENT OF EXP(-5.187809D 00)T*COS(2.532566D 00)T

1.0544D-03	2.4102D-04	-2.0381D 00	6.0784D-03	6.7190D-01	1.1217D-01	1.7412D-03	-1.2245D-02
2.5149D-05	-6.5480D-06	-8.7181D-02	4.6881D-04	-6.6591D-02	6.0420D-03	-6.1903D-05	-3.8627D-04
-4.5226D-04	-6.0365D-05	9.9510D-01	-3.7396D-03	6.8241D-04	-5.9887D-02	-3.8521D-04	5.5820D-03
1.1614D-04	2.4350D-05	-2.3066D-01	7.2728D-04	5.9279D-02	1.2956D-02	1.7336D-04	-1.3656D-03
1.9239D-04	1.7453D-04	-4.8937D-03	-2.3289D-03	9.9974D-01	-1.5273D-02	1.4159D-03	-1.2345D-03
3.7238D-05	4.1946D-06	-6.4119D-02	3.2838D-04	-5.2785D-03	5.1437D-03	2.5179D-05	-4.6556D-04
-1.0180D-03	-1.2562D-04	2.2689D 00	-8.6891D-03	7.0775D-02	-1.3762D-01	-7.8340D-04	1.2644D-02
7.2325D-05	-2.2248D-06	-1.9253D-01	9.1092D-04	-7.9935D-02	1.2830D-02	-3.8338D-05	-9.8367D-04

THE MATRIX COEFFICIENT OF EXP(-5.187809D 00)T*SIN(2.532566D 00)T

-3.2433D-04	1.8123D-04	-2.2546D-01	-9.5428D-04	9.2717D-01	5.1459D-03	1.4377D-03	-2.5454D-03
1.4008D-04	3.8361D-05	-2.5294D-01	6.4057D-04	1.3167D-01	1.3167D-02	2.8467D-04	-1.5763D-03
-6.0188D-04	-1.9661D-04	9.9694D-01	-1.9102D-03	-7.8148D-01	-4.7818D-02	-1.4922D-03	6.5367D-03
5.9322D-05	2.6048D-05	-7.9568D-02	1.3165D-05	1.2170D-01	2.8927D-03	2.0360D-04	-5.9334D-04
-1.3459D-03	-3.2824D-04	2.5437D 00	-7.2167D-03	-9.9661D-01	-1.3755D-01	-2.3959D-03	1.5473D-02
5.7922D-05	1.8356D-05	-9.7583D-02	1.9921D-04	7.1282D-02	4.7617D-03	1.3869D-04	-6.3354D-04
-1.4657D-03	-4.7153D-04	2.4495D 00	-4.8556D-03	-1.8510D 00	-1.1857D-01	-3.5686D-03	1.5978D-02
2.2412D-04	6.4358D-05	-3.9633D-01	9.4639D-04	2.3101D-01	2.0250D-02	4.8054D-04	-2.5024D-03

THE COMPLEX EIGENVECTOR IS

MAGNITUDE	PHASE (DEG)
1.1031080D-03	-1.7098202D 01
1.4231637D-04	-7.9821576D 01
7.5286370D-04	1.2852195D 02
1.3041172D-04	-2.7057565D 01
1.3595433D-03	8.1864779D 01
6.8859886D-05	-5.7203149D 01
1.7845529D-03	1.2476230D 02
2.3550130D-04	-7.2114735D 01

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THE GAIN MATRIX G

0.0	0.0	4.5000D-01	8.5000D-01	0.0	0.0	0.0	0.0

THE MODIFIED PLANT MATRIX, A-FG

-1.7090D-03	-1.4430D-03	8.1348D 00	-2.1188D 01	7.6930D-01	-2.7350D-01	2.2310D-03	0.0
-5.0330D-03	-2.3660D-01	3.7720D 00	7.3925D 00	6.7200D-01	6.5740D-01	-1.7380D-02	0.0
4.2927D-04	-3.1960D-04	-6.8522D 00	-7.9526D 00	-1.9810D 00	1.6880D-01	-1.0340D-03	0.0
0.0	0.0	1.0000D 00	0.0	0.0	0.0	0.0	0.0
-4.3990D-03	-1.7060D-03	5.9352D 00	-9.8940D-01	-7.7130D 00	-2.7480D-01	-1.3190D-02	0.0
3.5200D-05	-1.9300D-03	2.5165D-01	2.2695D-01	2.0490D-01	-3.6610D-01	1.2570D-02	0.0
-2.0400D-03	-1.4240D-02	-1.4786D 00	-4.1625D 00	-2.4770D 00	5.6080D-01	-2.6050D-02	3.2200D 01
0.0	0.0	0.0	0.0	1.0000D 00	0.0	0.0	0.0

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THE CHARACTERISTIC POLYNOMIAL - IN ASCENDING POWERS OF S

1.0189036D-04	5.2586636D-01	3.3933966D 00	1.1617375D 01	4.8006537D 01	1.0634633D 02
8.1822891D 01	1.5195849D 01	1.0000000D 00			

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THE EIGENVALUES OF THE A MATRIX

REAL PART	IMAGINARY PART
-1.9399244D-04	0.0
-3.8195892D-02	-2.7676314D-01
-3.8195892D-02	2.7676314D-01
-2.4473216D-01	0.0
-4.7775410D-01	0.0
-9.7459679D-01	0.0
-6.7874819D 00	-3.6027406D 00
-6.7874819D 00	3.6027406D 00

****

THE ELEMENTS OF THE STATE TRANSITION MATRIX

THE MATRIX COEFFICIENT OF EXP(-1.939924D-04)T

9.9216D-01	1.3656D-02	-2.5614D 00	-2.0309D 01	-8.5030D-01	-1.2600D 00	7.3178D-05	-1.2146D 01
-2.4953D-02	-3.4345D-04	6.4422D-02	5.1080D-01	2.1386D-02	3.1690D-02	-1.8405D-06	3.0549D-01
-1.3013D-09	1.8062D-11	-3.3619D-09	-2.6680D-08	-1.3099D-09	-1.6538D-09	-1.7053D-13	-1.5929D-08
-6.7072D-06	-9.2316D-08	1.7316D-05	1.3733D-04	5.7462D-06	8.5179D-06	-4.9475D-10	8.2113D-05
-1.4606D-09	-2.0077D-11	4.1842D-09	3.0641D-08	1.0978D-09	1.8894D-09	5.6843D-14	1.7852D-08
-6.4219D-03	-8.8390D-05	1.6579D-02	1.3146D-01	5.5037D-03	8.1556D-03	-4.7366D-07	7.8621D-02
-1.9337D-01	-2.6615D-03	4.9921D-01	3.9583D 00	1.6572D-01	2.4557D-01	-1.4262D-05	2.3673D 00
7.5290D-06	1.0363D-07	-1.9438D-05	-1.5412D-04	-6.4525D-06	-9.5616D-06	5.5568D-10	-9.2176D-05

THE MATRIX COEFFICIENT OF EXP(3.819589D-02)T*COS(2.767631D-01)T

9.7865D-03	-6.2393D-03	3.9117D-02	-5.6408D 00	9.9411D-01	-9.0151D-02	5.4526D-02	8.1437D 00
-3.8635D-03	2.0767D-03	-1.9440D-02	1.7697D 00	-3.1902D-01	9.2747D-03	-2.0564D-02	-2.6428D 00
3.6521D-05	-1.5759D-05	2.2340D-04	-1.2195D-02	2.2874D-03	1.7328D-04	1.8484D-04	1.9307D-02
1.6681D-04	2.6133D-05	2.0371D-03	6.0582D-02	-8.2309D-03	7.4850D-03	5.9961D-04	-5.7357D-02
-4.9975D-04	-4.7873D-07	-5.2981D-03	-8.9440D-02	9.8720D-03	-1.7125D-02	-1.5900D-03	5.6614D-02
4.5919D-03	4.7469D-04	5.3546D-02	1.3782D 00	-1.8006D-01	1.8938D-01	1.7115D-02	-1.2166D 00
2.0102D-01	-2.0357D-02	1.9182D 00	1.1619D 01	-5.8352D-02	5.4865D 00	8.5174D-01	7.7139D 00
6.8289D-05	-5.9760D-04	-5.4520D-03	-6.9412D-01	1.1211D-01	-3.8318D-02	-1.7582D-03	8.7634D-01

THE MATRIX COEFFICIENT OF EXP(3.819589D-02)T*SIN(2.767631D-01)T

-1.7395D-02	-2.2111D-03	-2.0711D-01	-5.7092D 00	7.6062D-01	-7.4551D-01	-6.3804D-02	5.2199D 00
5.6844D-03	9.3959D-04	6.9930D-02	2.1229D 00	-2.8988D-01	2.5843D-01	2.0309D-02	-2.0277D 00
-4.2006D-05	-9.5509D-06	-5.4371D-04	-1.8769D-02	2.6371D-03	-2.0871D-03	-1.4360D-04	1.8841D-02
1.0894D-04	-6.0650D-05	5.2604D-04	-5.2424D-02	9.4008D-03	-4.0690D-04	5.8513D-04	7.7677D-02
-8.8230D-05	1.6831D-04	8.0647D-04	1.8342D-01	-3.0257D-02	8.4437D-03	-7.7052D-04	-2.3935D-01
2.2499D-03	-1.6261D-03	6.9886D-03	-1.5260D 00	2.6536D-01	-3.3920D-02	1.3018D-02	2.1551D 00
-2.7519D-02	-6.4127D-02	-9.5478D-01	-8.0757D 01	1.2725D 01	-5.3080D 00	4.9983D-02	9.8031D 01
-1.8151D-03	8.0662D-05	-1.8391D-02	-2.2737D-01	2.0197D-02	-5.6589D-02	-7.4331D-03	8.3616D-02

THE COMPLEX EIGENVECTOR IS

MAGNITUDE	PHASE (DEG)
1.9958628D-02	6.0637006D 01
6.8730464D-03	-1.2420271D 02
5.5662504D-05	4.8995913D 01
1.9923126D-04	-3.3148402D 01
5.0747514D-04	1.6998774D 02
5.1134442D-03	-2.6103421D 01
2.0289519D-01	7.7950063D 00
1.8163917D-03	8.7845422D 01

THE MATRIX COEFFICIENT OF EXP(-2.447322D-01)T

-1.0525D-03	-2.9737D-02	-1.8037D-02	2.1689D-01	-3.1359D-02	-1.0485D-01	1.6580D-02	-2.2341D-01
3.7895D-02	1.0312D 00	6.2548D-01	-7.5212D 00	1.0874D 00	3.6360D 00	-5.8882D-02	7.7473D 00
2.4881D-06	6.7726D-05	4.1079D-05	-4.9396D-04	7.1418D-05	2.3880D-04	-3.8671D-06	5.0881D-04
-1.0167D-05	-2.7673D-04	-1.6785D-04	2.0184D-03	-2.9182D-04	-9.7575D-04	1.5802D-05	-2.0790D-03
1.9978D-07	5.4380D-06	3.2978D-06	-3.9661D-05	5.7347D-06	1.9174D-05	-3.1051D-07	4.0855D-05
-2.8294D-04	-7.7014D-03	-4.6713D-03	5.6171D-02	-8.1213D-03	-2.7155D-02	4.3975D-04	-5.7859D-02
3.1281D-03	8.5146D-02	5.1645D-02	-6.2102D-01	8.9788D-02	3.0022D-01	-4.8619D-03	6.3969D-01
-9.1633D-07	-2.2220D-05	-1.3478D-05	1.6206D-04	-2.3432D-05	-7.8347D-05	1.2688D-06	-1.6694D-04

THE MATRIX COEFFICIENT OF EXP(-4.777541D-01)T

9.3749D-03	2.8567D-02	-6.1430D-01	-1.0545D 01	9.2475D-01	3.4008D 00	-6.7986D-02	4.5822D 00
-1.1197D-02	-3.4142D-02	7.3367D-01	1.2594D 01	-1.1044D 00	-4.0616D 00	8.1158D-02	-5.4726D 00
-6.9552D-05	-2.1209D-04	4.5575D-03	7.8232D-02	-6.8607D-03	-2.5231D-02	5.0439D-04	-3.3995D-02
1.4558D-04	4.4392D-04	-9.5393D-03	-1.6375D-01	1.4360D-02	5.2811D-02	-1.0556D-03	7.1157D-02
-1.3242D-04	-4.0379D-04	8.6770D-03	1.4895D-01	-1.3062D-02	-4.8036D-02	9.6031D-04	-6.4724D-02
2.4715D-03	7.5364D-03	-1.6195D-01	-2.7800D 00	2.4380D-01	8.9656D-01	-1.7924D-02	1.2080D 00
-2.2750D-02	-6.9371D-02	1.4907D 00	2.5589D 01	-2.2441D 00	-8.2526D 00	1.6498D-01	-1.1120D 01
2.7717D-04	8.4518D-04	-1.8162D-02	-3.1176D-01	2.7341D-02	1.0055D-01	-2.0101D-03	1.3547D-01

THE MATRIX COEFFICIENT OF EXP(-9.745368D-01)T

-1.0789D-02	-6.4067D-03	4.7277D 00	3.7371D 01	-1.5084D 00	-2.0004D 00	1.0595D-02	-3.5005D-01
1.9782D-03	1.1747D-03	-8.6684D-01	-6.8522D 00	2.7657D-01	3.6678D-01	-1.9427D-03	6.4184D-02
-3.3818D-04	-2.0082D-04	1.4819D-01	1.1714D 00	-4.7282D-02	-6.2704D-02	3.3211D-04	-1.0973D-02
-3.4700D-04	-2.0606D-04	1.5206D-01	1.2020D 00	-4.8515D-02	-6.4338D-02	3.4077D-04	-1.1259D-02
3.4666D-04	2.0586D-04	-1.5191D-01	-1.2008D 00	4.8467D-02	6.4275D-02	-3.4043D-04	1.1248D-02
-3.7262D-04	-2.2127D-04	1.6328D-01	1.2907D 00	-5.2097D-02	-6.9089D-02	3.6593D-04	-1.2090D-02
1.2211D-02	7.2513D-03	-5.3509D 00	-4.2298D 01	1.7073D 00	2.2641D 00	-1.1992D-02	3.9620D-01
-3.5569D-04	-2.1122D-04	1.5587D-01	1.2321D 00	-4.9731D-02	-6.5950D-02	3.4931D-04	-1.1541D-02

THE MATRIX COEFFICIENT OF EXP(-6.787482D 00)T*COS(3.602741D 00)T

5.6293D-04	1.4103D-04	-1.5730D 00	-1.0930D 00	4.7123D-01	5.4608D-02	1.0942D-03	-5.9558D-03
1.5065D-04	2.2920D-05	-5.3729D-01	-5.0100D-01	3.8076D-02	1.7911D-02	1.5251D-04	-1.5741D-03
-3.0764D-04	-4.0666D-05	1.1434D 00	1.1059D 00	-4.2781D-02	-3.7885D-02	-3.5326D-04	3.2066D-03
5.1478D-05	1.2830D-05	-1.4440D-01	-1.0095D-01	4.2672D-02	5.0094D-03	5.5685D-05	-5.4454D-04
2.8531D-04	1.9297D-04	1.4852D-01	1.1413D 00	9.5472D-01	8.6749D-04	1.3705D-03	-3.1793D-03
1.4073D-05	5.2003D-10	-6.6787D-02	-7.6548D-02	-9.0042D-03	2.1436D-03	3.6680D-06	-1.4423D-04
-2.4165D-04	3.1385D-05	1.3912D 00	1.7525D 00	3.3963D-01	-1.3736D-02	1.4789D-04	2.4350D-03
-3.5189D-06	-1.4326D-05	-1.2222D-01	-2.2621D-01	-8.9651D-02	3.9105D-03	-9.8737D-05	-1.6427D-05

THE MATRIX COEFFICIENT OF EXP(-6.787482D 00)T*SIN(3.602741D 00)T

1.2255D-04	1.3957D-04	5.0595D-01	1.2828D 00	7.4474D-01	-1.2161D-02	9.7007D-04	-1.4407D-03
1.1363D-04	6.0868D-05	-6.5140D-02	2.3173D-01	2.8614D-01	3.8689D-03	4.3857D-04	-1.2451D-03
-2.6414D-04	-1.3368D-04	2.1270D-01	-4.2389D-01	-6.1980D-01	-1.0730D-02	-9.0668D-04	2.8839D-03
1.1592D-05	1.2884D-05	4.5310D-02	1.1677D-01	6.8519D-02	-1.0781D-03	8.9563D-05	-1.3585D-04
-5.9519D-04	-1.2039D-04	1.8873D 00	1.5574D 00	-3.2361D-01	-5.4089D-02	-9.6363D-04	6.2590D-03
2.2148D-05	9.0424D-06	-3.4701D-02	5.7294D-03	3.9115D-02	1.3778D-03	6.6505D-05	-2.3894D-04
-5.5018D-04	-2.0468D-04	1.0173D 00	1.3610D-01	-8.5418D-01	-3.8629D-02	-1.5181D-03	5.9953D-03
8.5822D-05	2.5612D-05	-2.0787D-01	-1.0938D-01	9.6021D-02	7.4197D-03	1.9438D-04	-9.1343D-04

THE COMPLEX EIGENVECTOR IS

MAGNITUDE	PHASE (DEG)
5.7611456D-04	-1.2281311D 01
1.8859893D-04	-3.7024672D 01
4.0547879D-04	1.3535057D 02
5.2766644D-05	-1.2690485D 01
6.6034910D-04	6.4388740D 01
2.6240864D-05	-5.7566600D 01
6.0090736D-04	1.1371237D 02
8.5893763D-05	-9.7652312D 01

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Thesis 169546
C514 Cline
c.1 A stability analysis
of the proposed circulation control rotor (CCR)
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